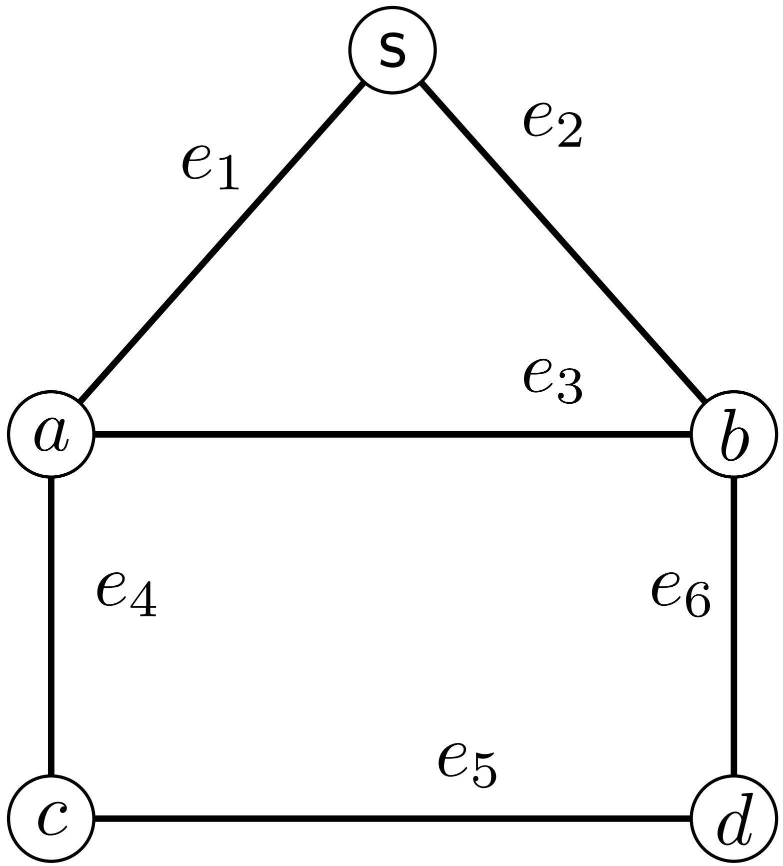
One-parameter Mechanisms

The private-edge Shortest-Paths Tree (SPT) problem



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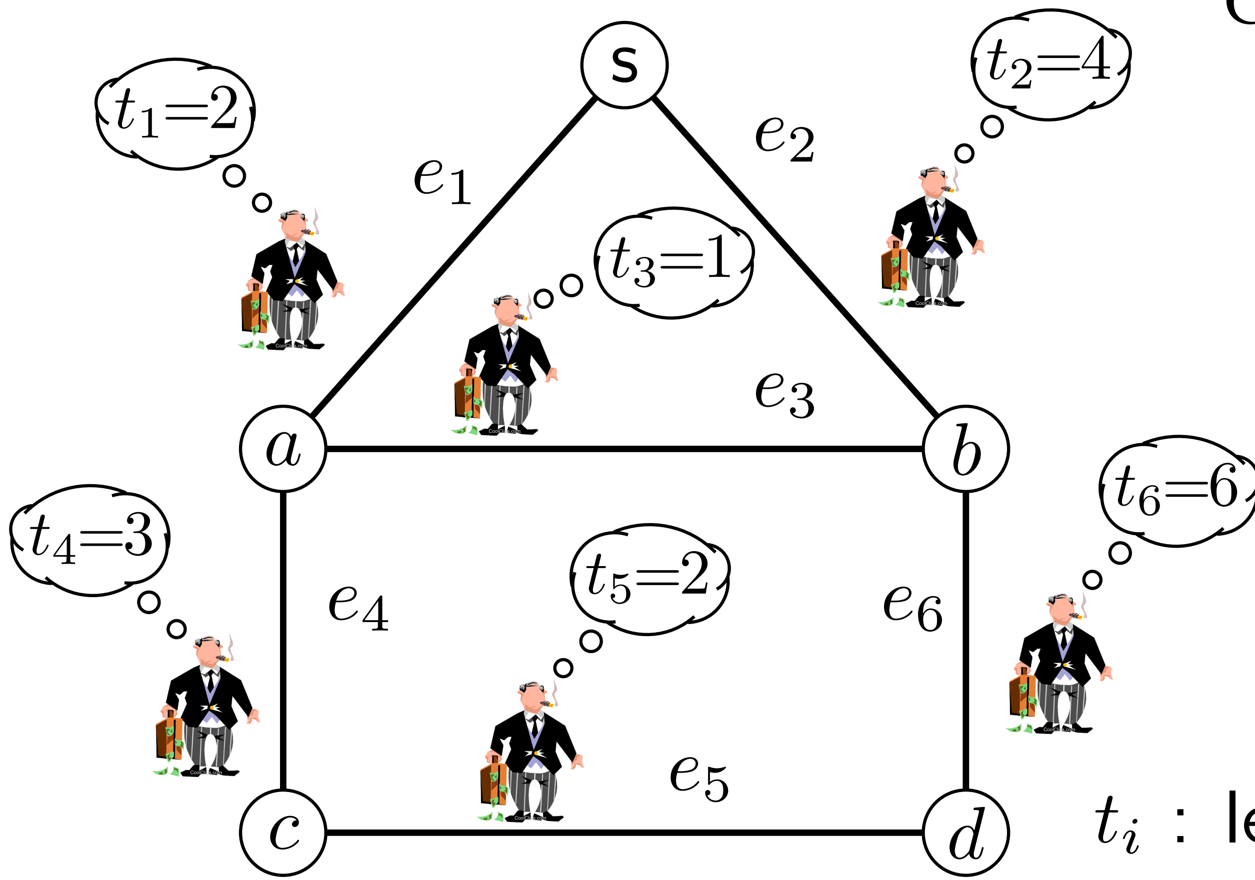

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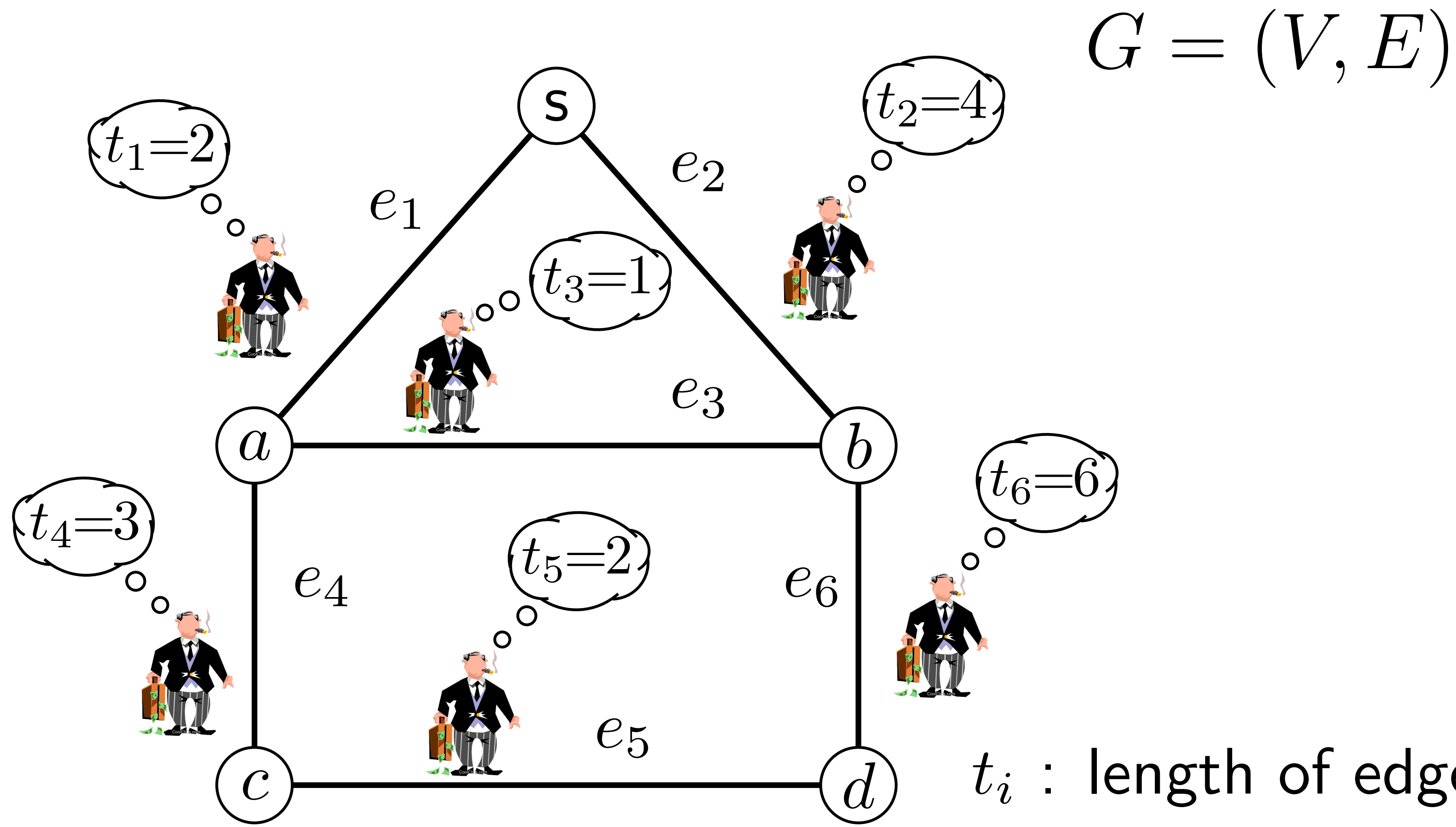

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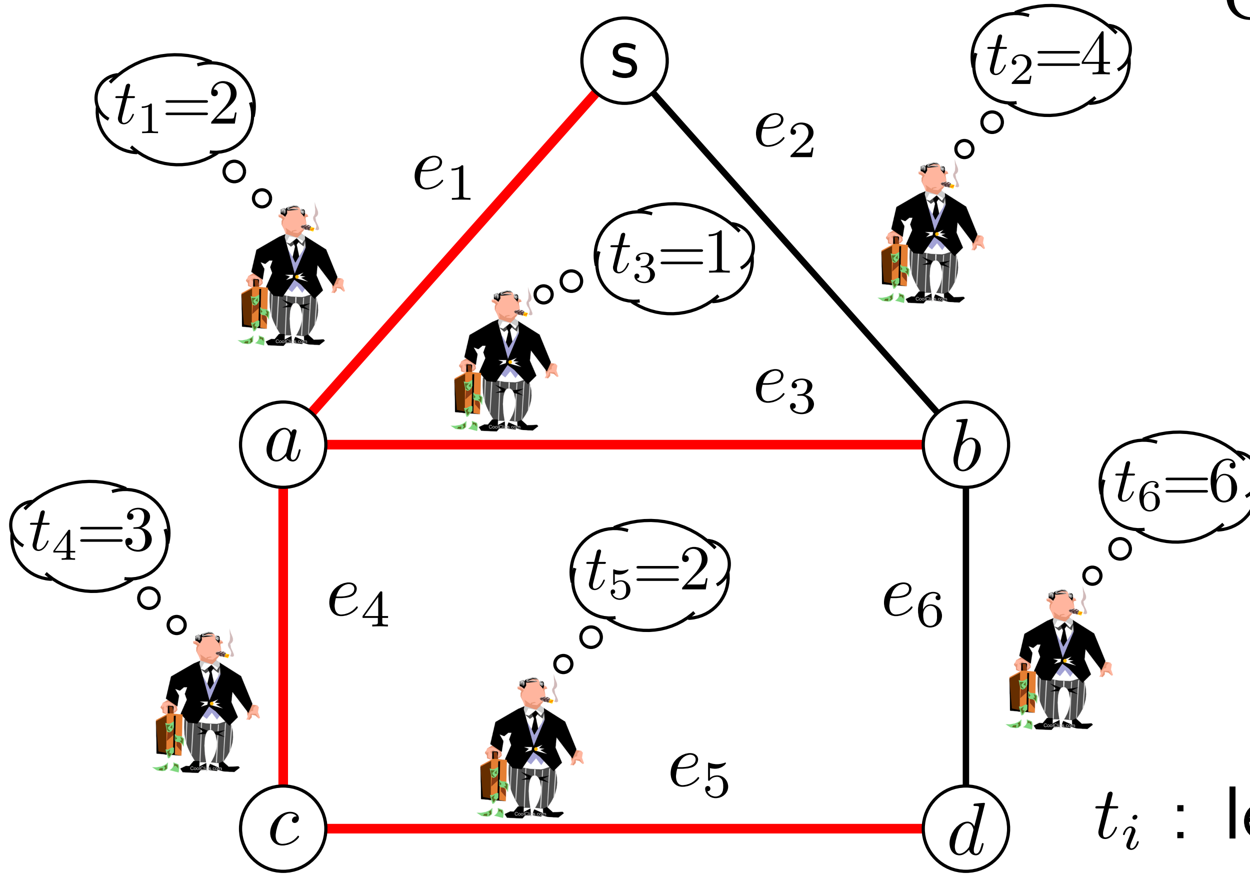

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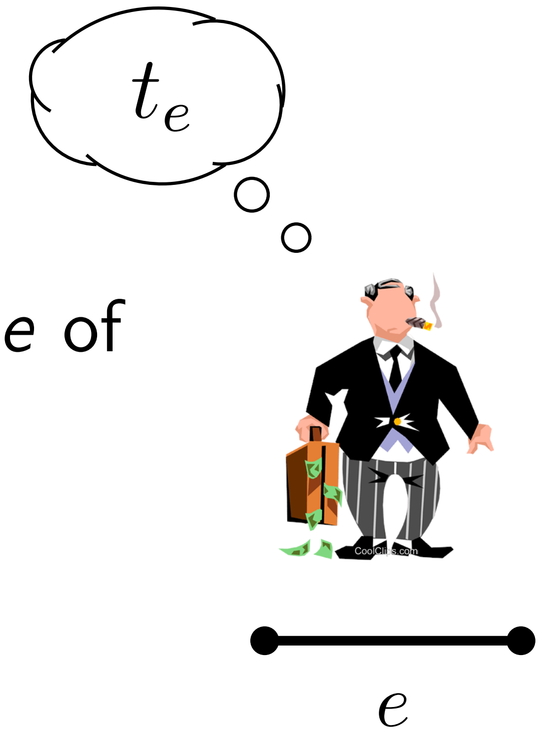

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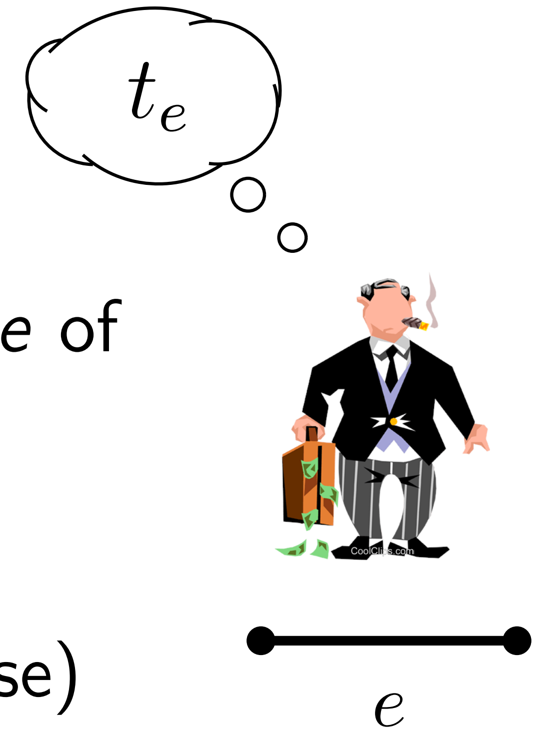

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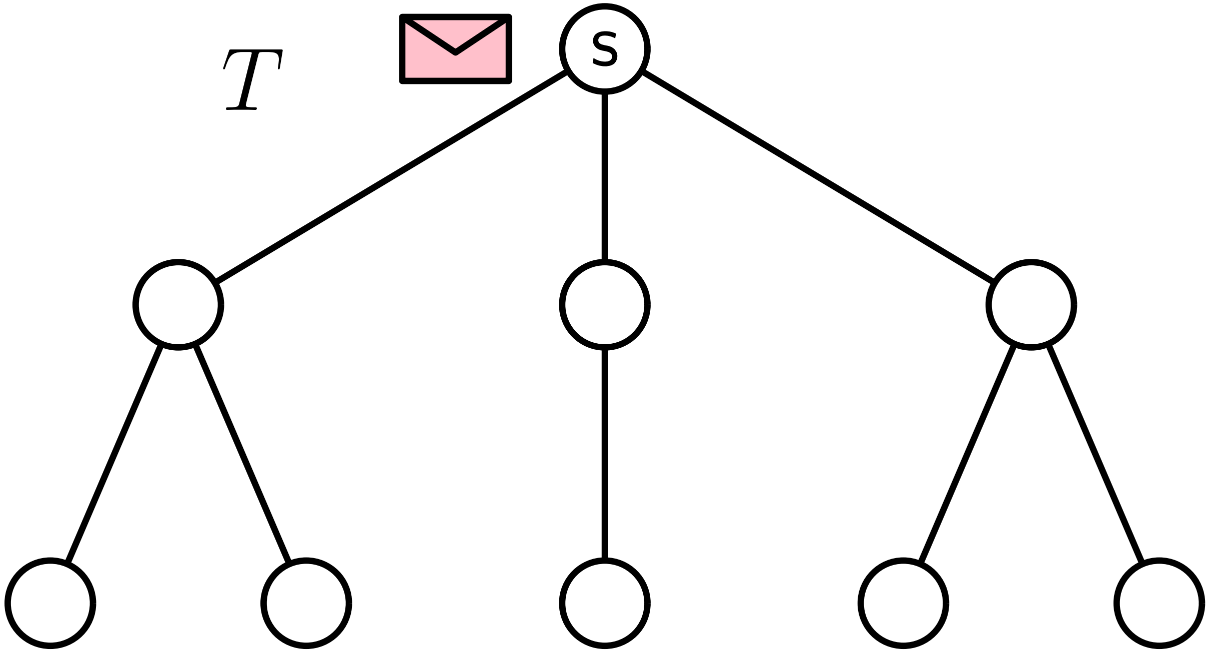
- The valuation of agent e w.r.t. to a tree T is:

$$v_e(t_e, T) = \begin{cases} t_e & \text{if } e \in E(T) \\ 0 & \text{if } e \notin E(T) \end{cases}$$

Note: v_e represents a **cost** incurred by agent e!

This models the multicast protocol



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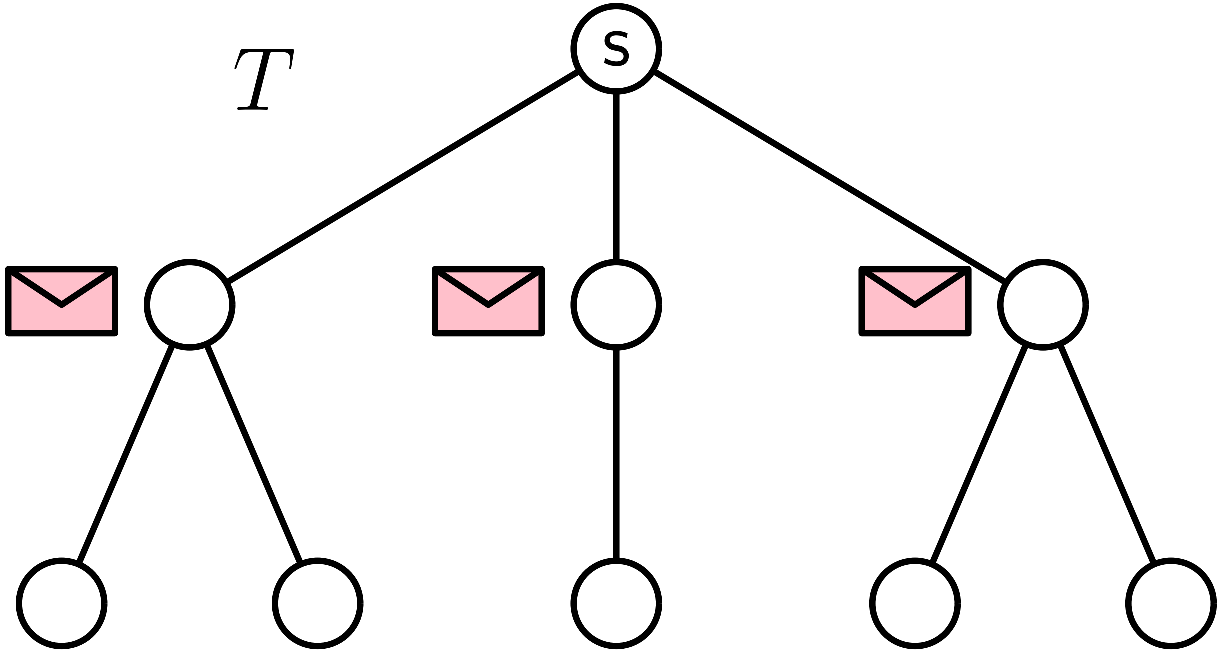

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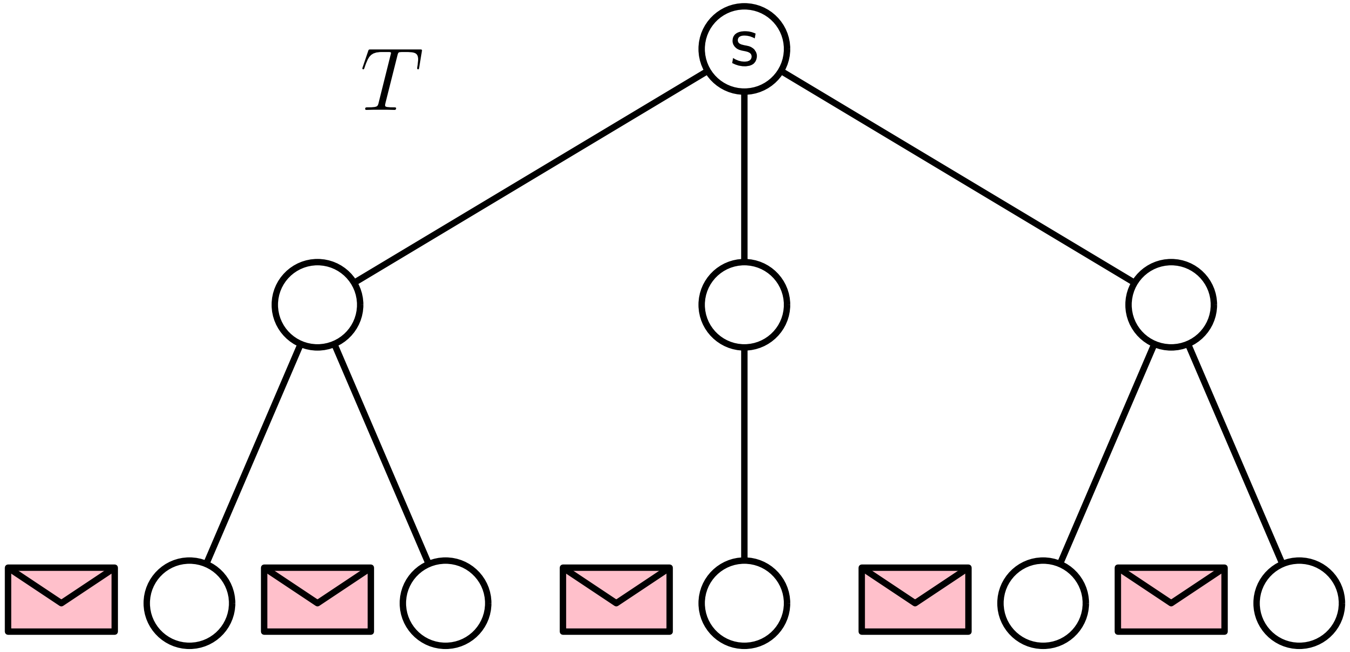

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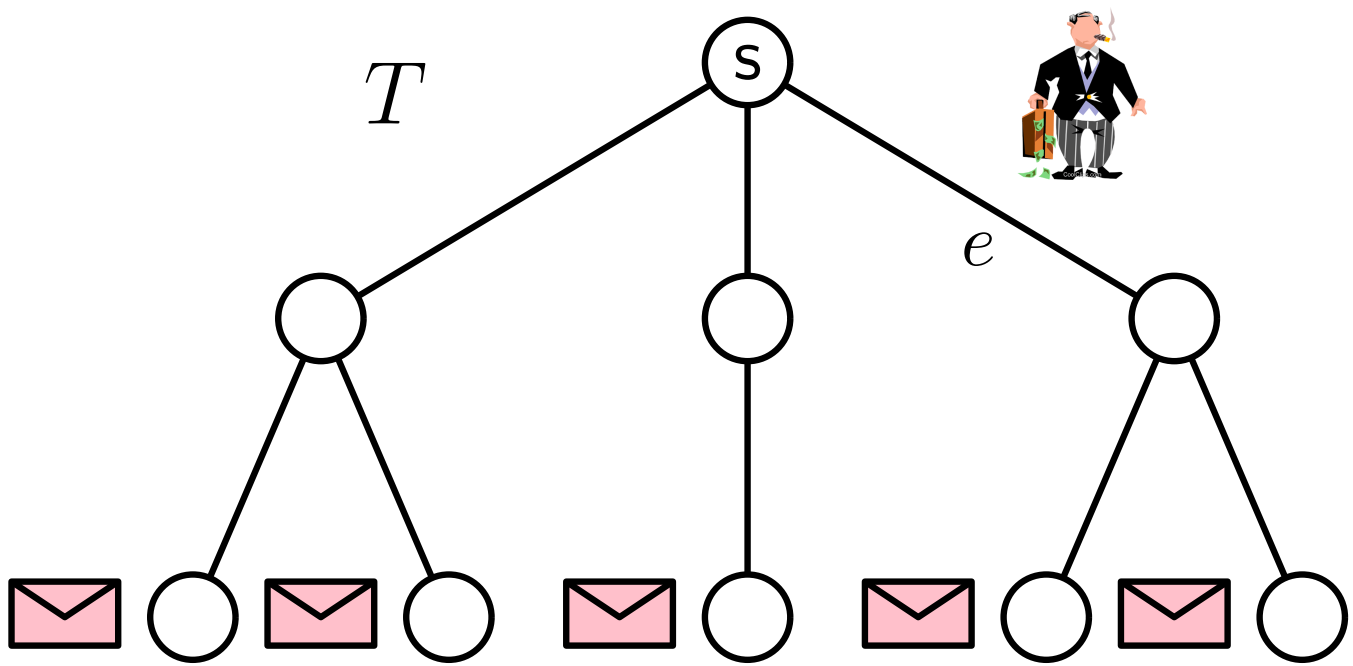

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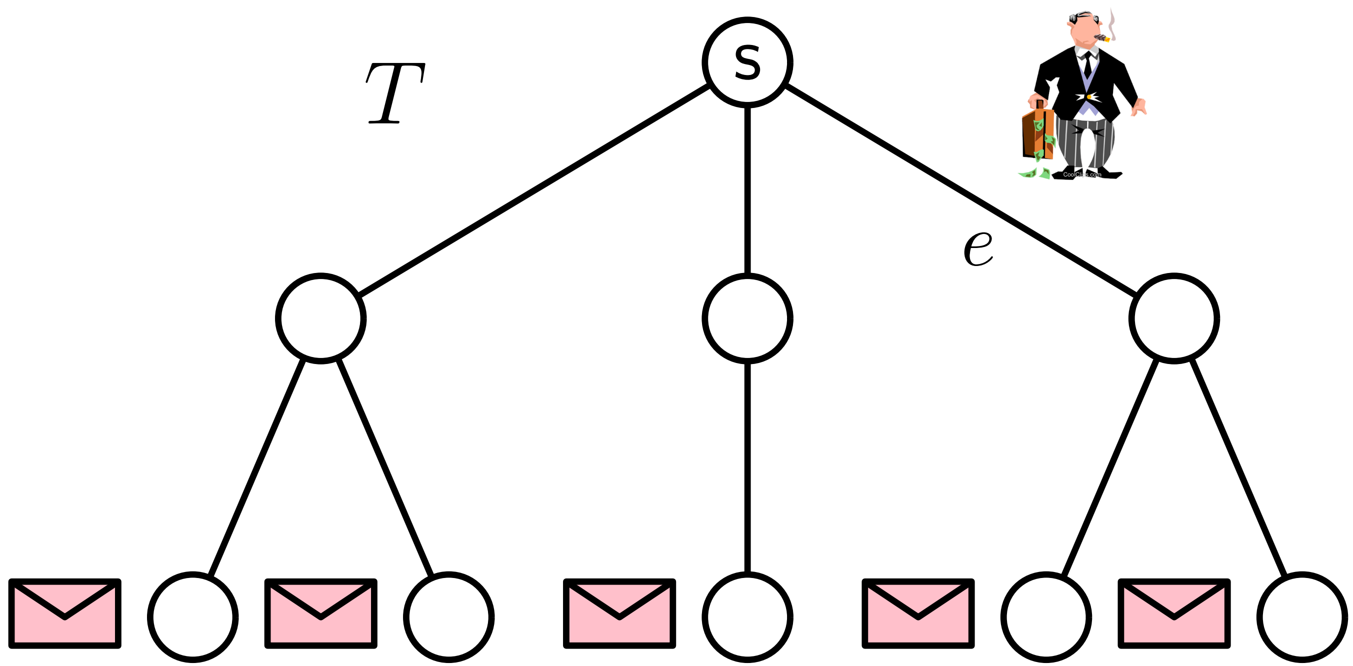

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We want to minimize the time needed to deliver the message from s to each node: T must be a SPT.

The private-edge Shortest-Paths Tree (SPT) problem

Goal:

- Let $\mathcal{F}$ be the set of all spanning trees of G rooted at s
- We want to design a truthful mechanism that **minimizes** the following quantity w.r.t $T \in \mathcal{F}$:

$$f(t,T) = \sum_{v \in V} d_T(s,v) = \sum_{e \in E(T)} t_e \cdot \|e\|,$$

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Note: **Non-utilitarian problem!**

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A necessary condition for designing OP truthful mechanisms

Definition: An algorithm g for a minimization OP problem is *monotone* if, $\forall$ player i, and $\forall r_{-i} = (r_1, \ldots, r_{i-1}, r_{i+1}, r_N)$ it holds that:

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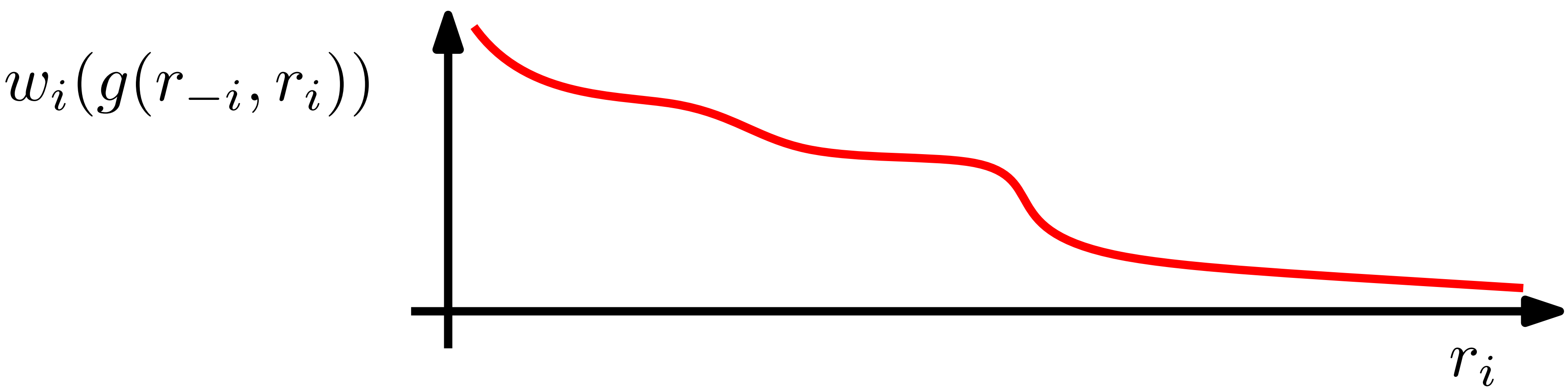

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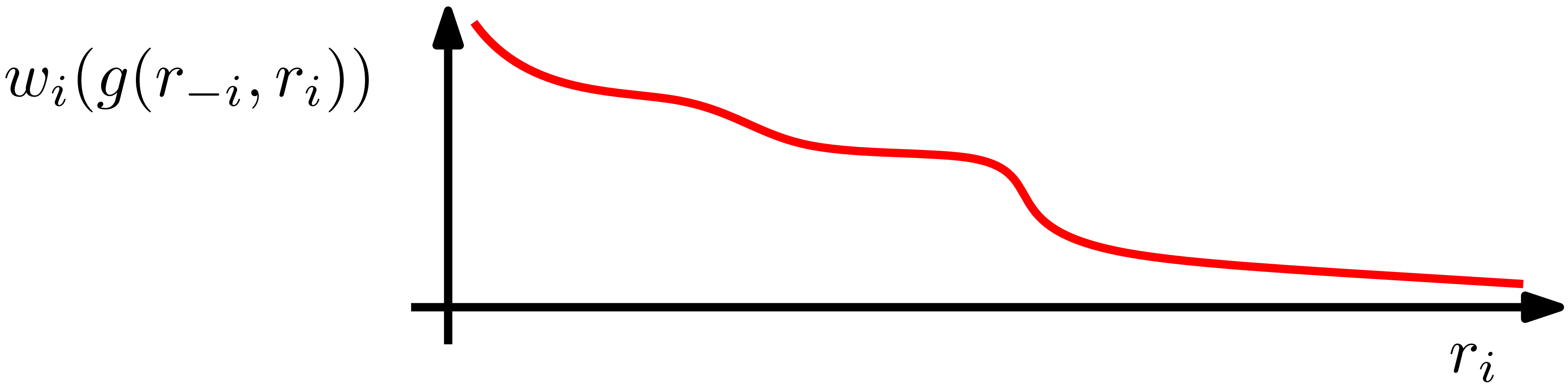

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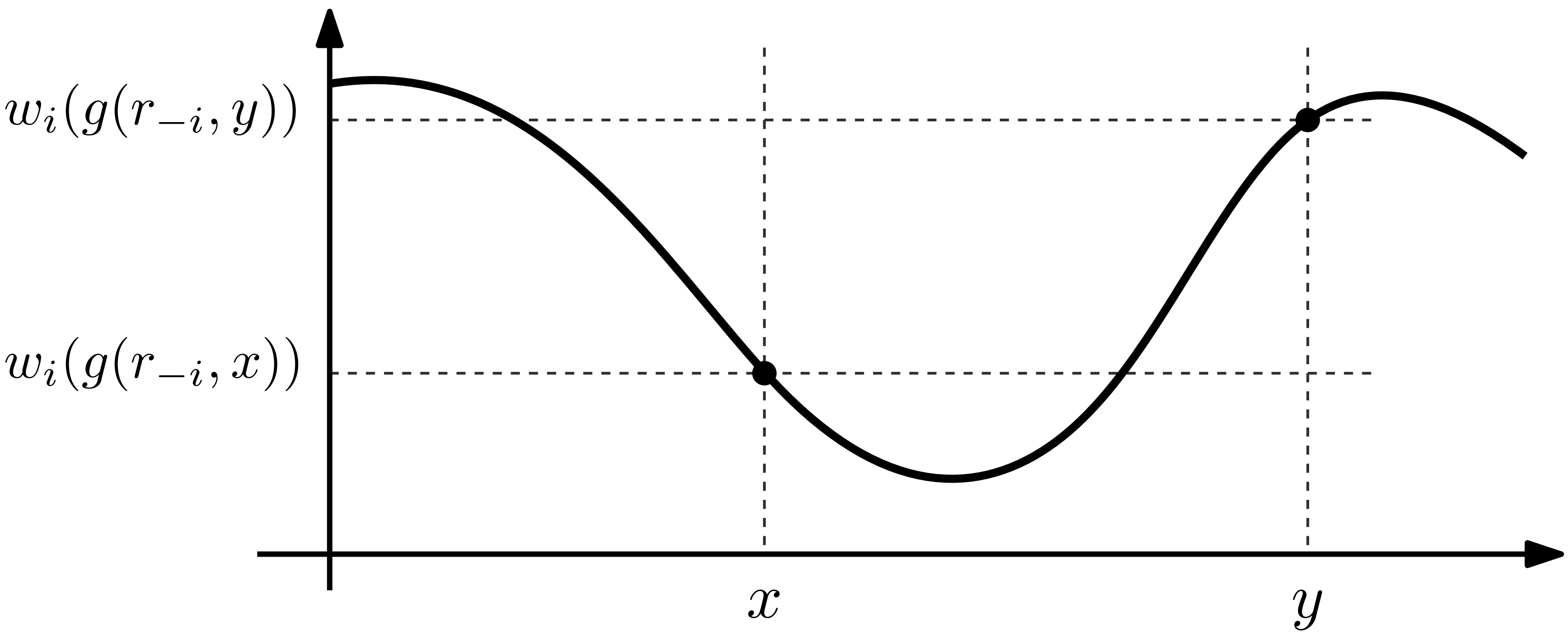
- There exists $x, y \in \mathbb{R}$ such that $x < y$ and $w_i(g(r_{-i}, x)) < w_i(g(r_{-i}, y))$

Proof (cont.):

Consider $t_i = x$:



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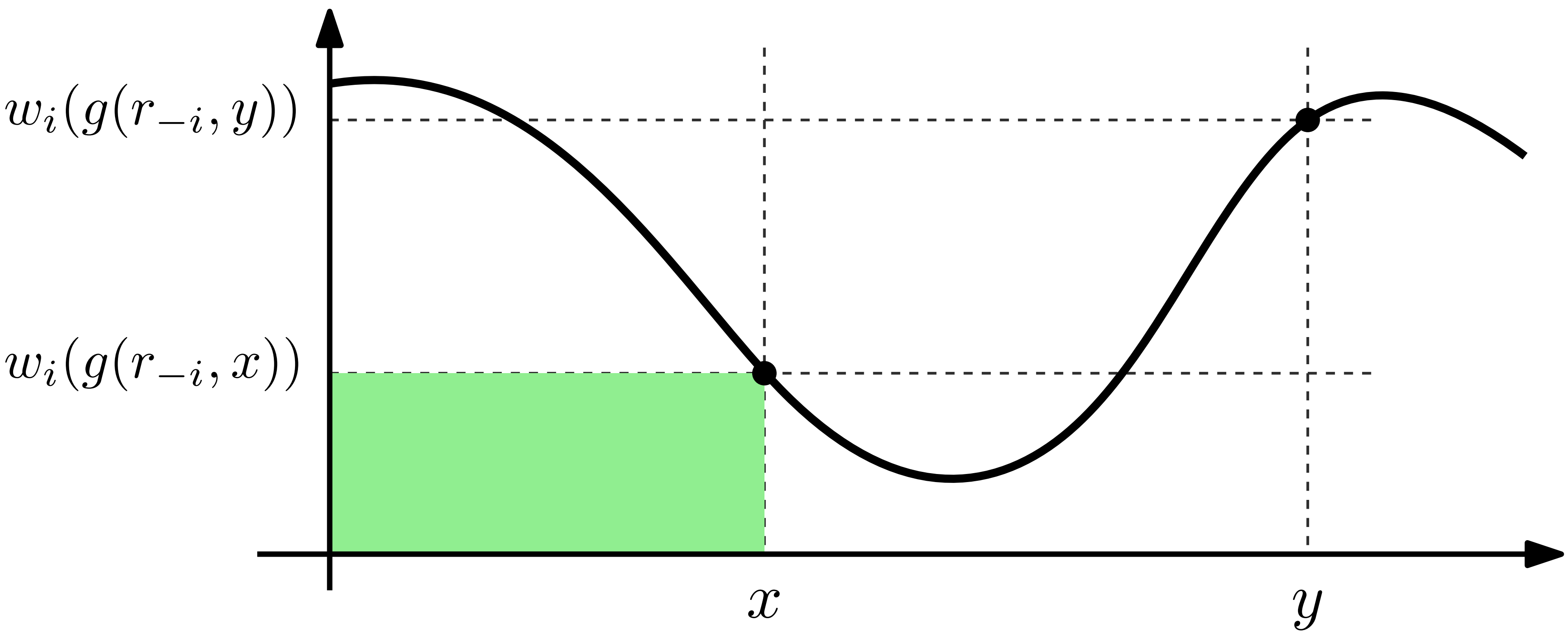

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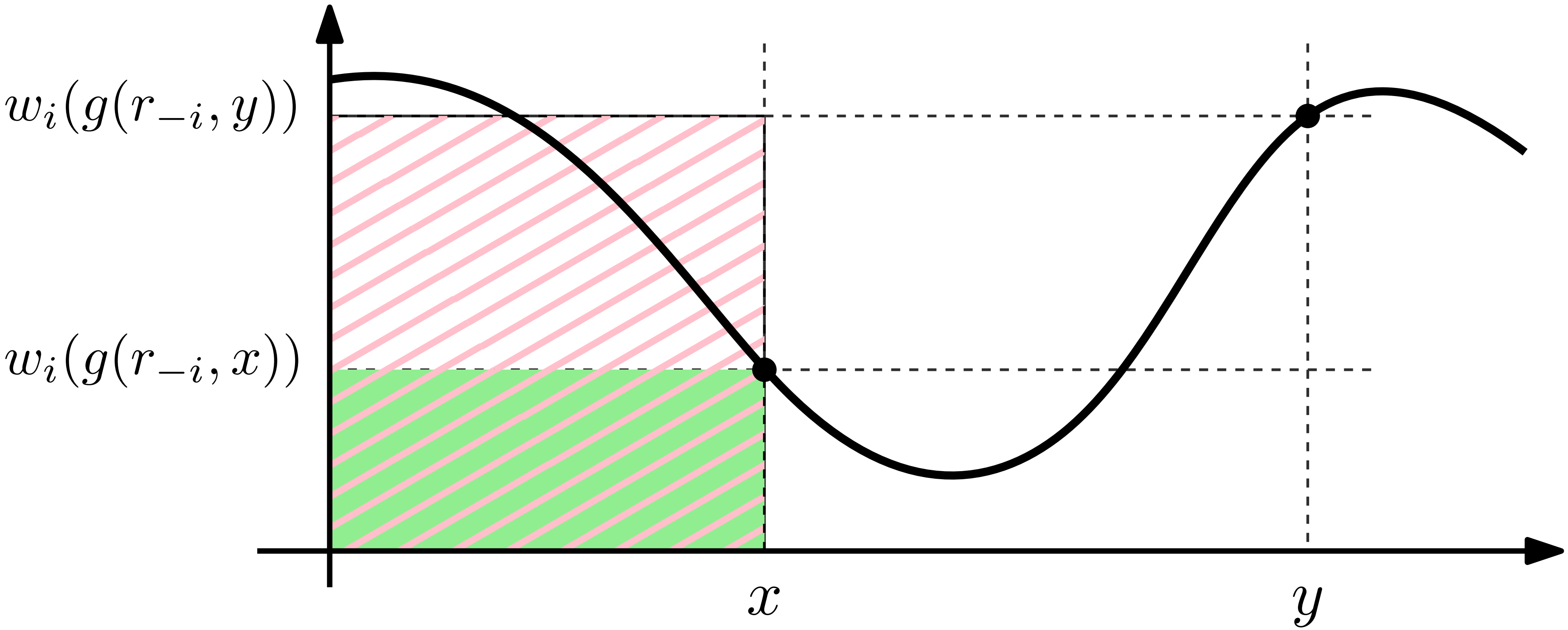

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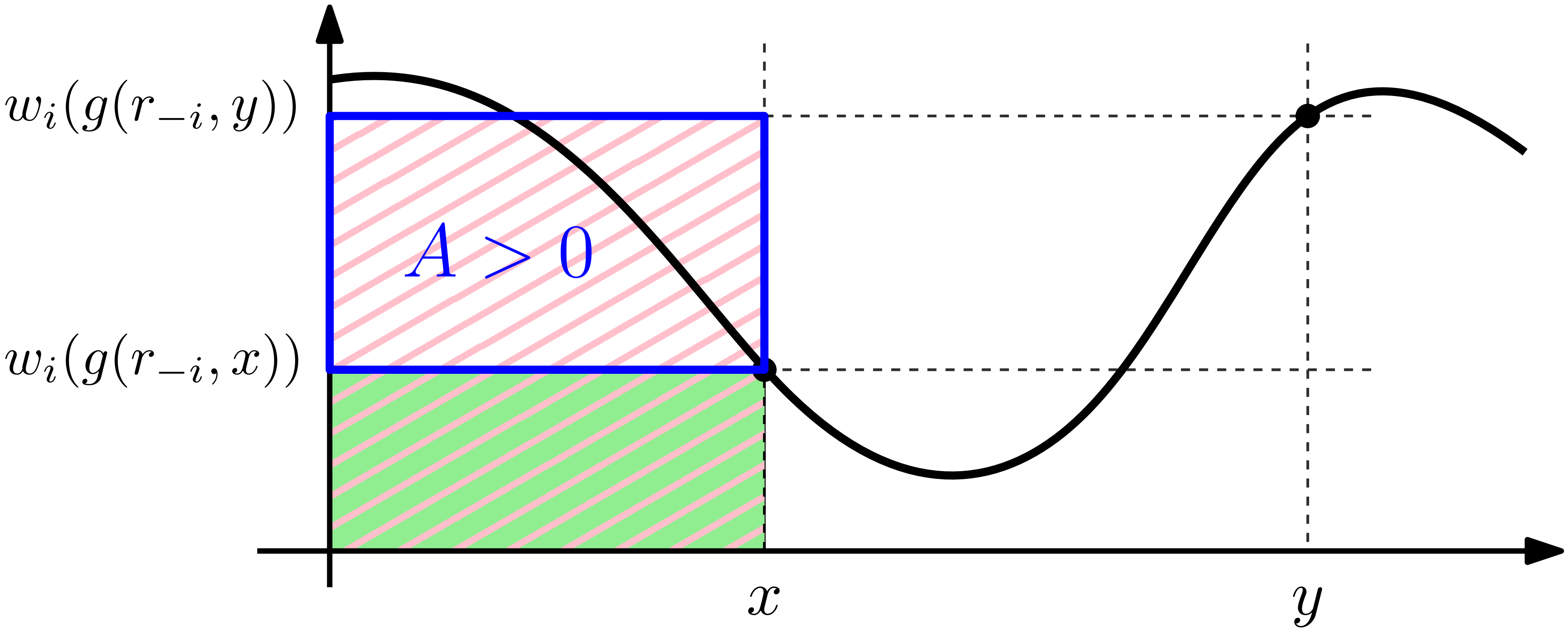

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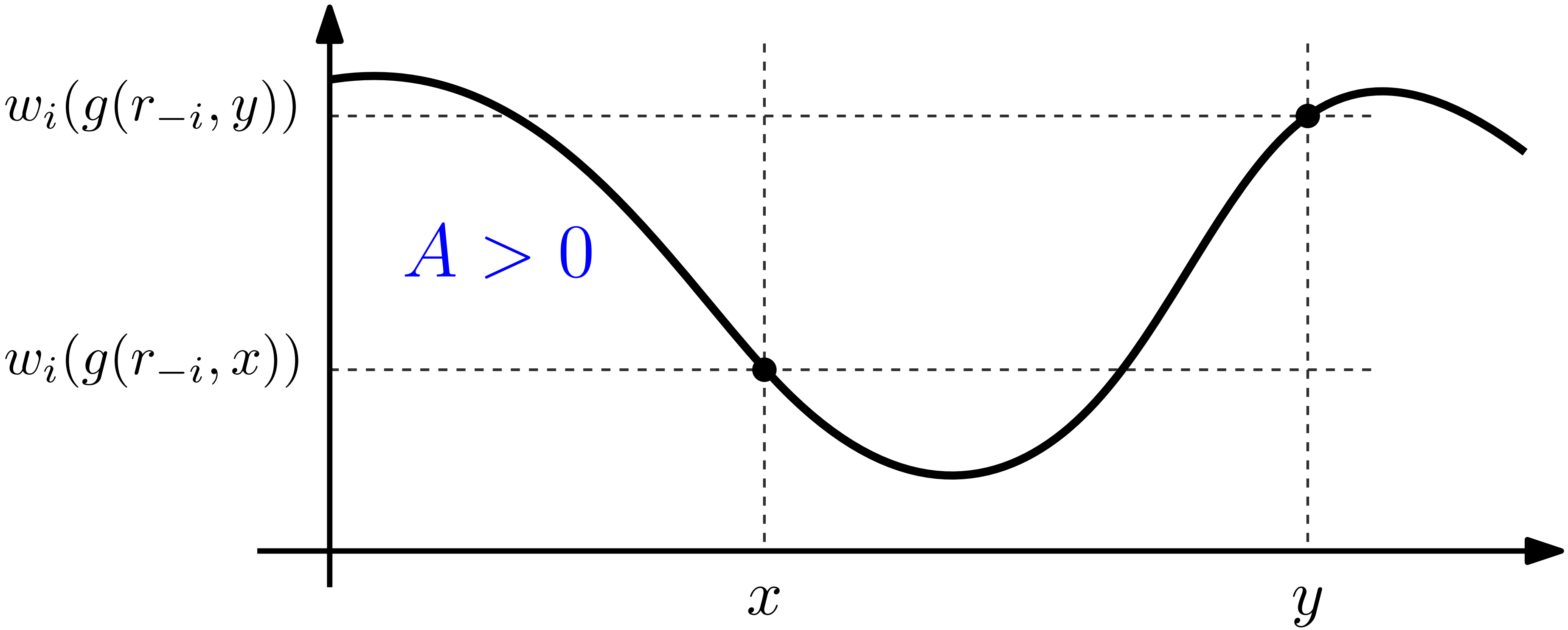

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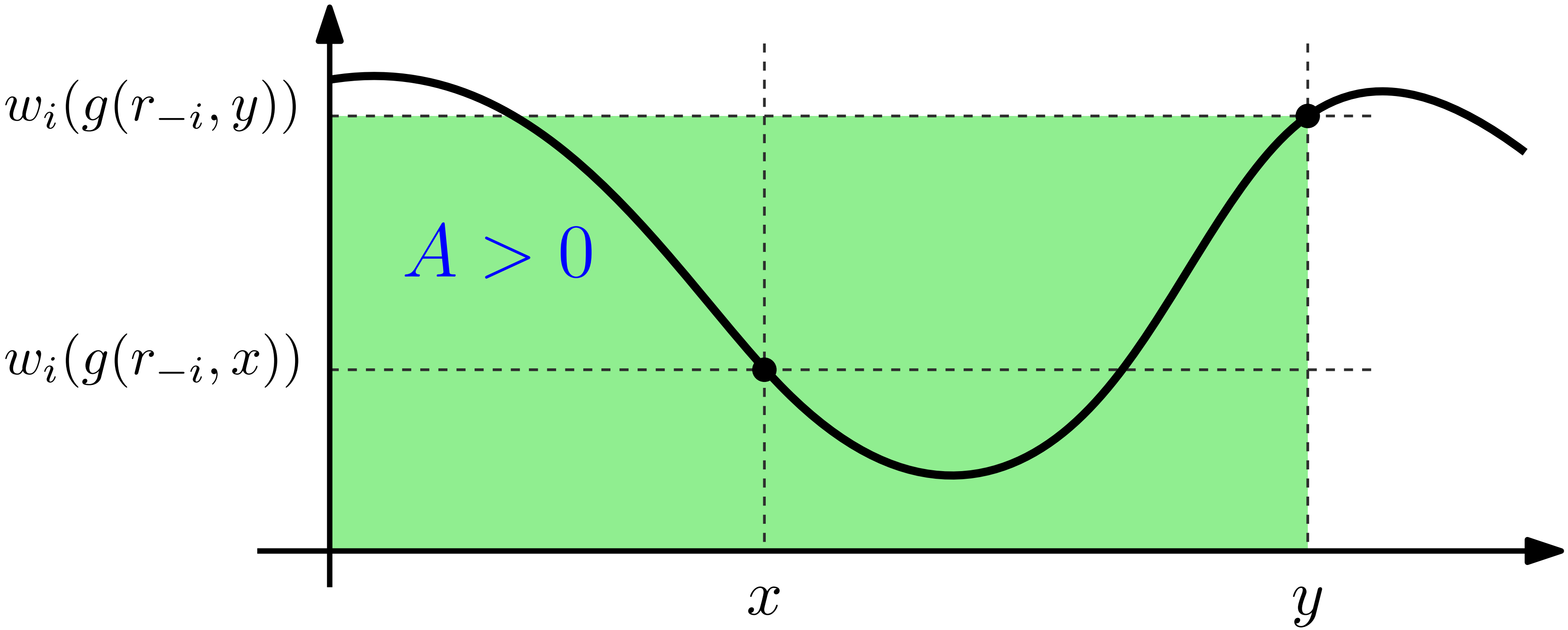

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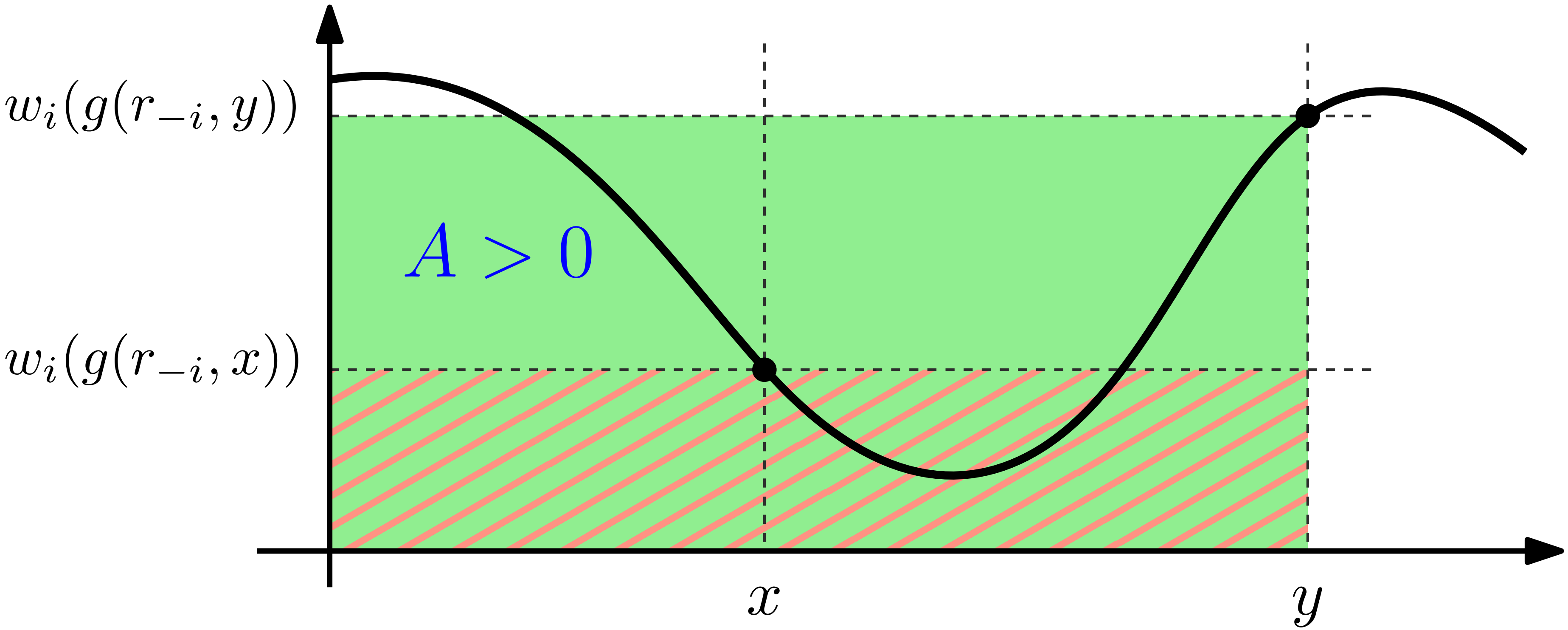

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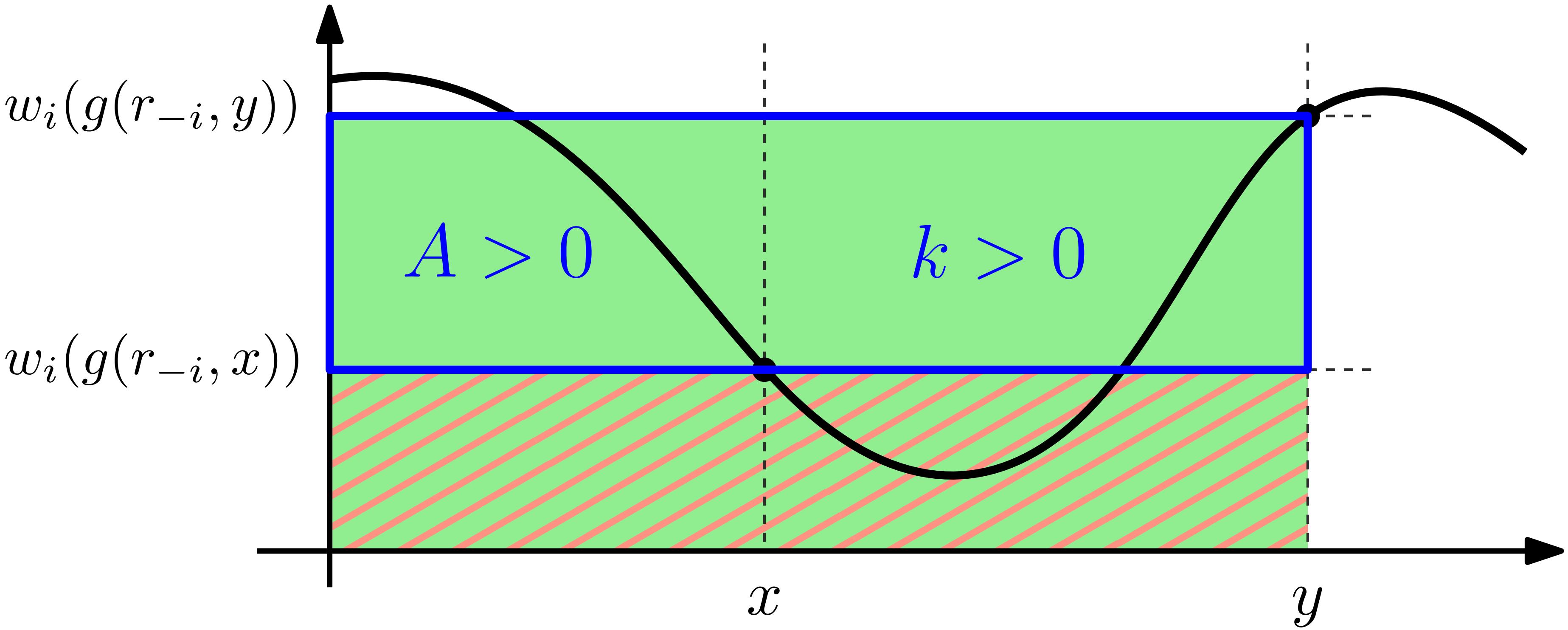

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If $\Delta p > A$ then player i has an incentive to lie when $t_i = x$.

(report y: cost increases by A, payment increases by $\Delta p > A$)

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$\square$

One-parameter Mechanisms

A one-parameter (OP) mechanism (for a OP problem) is a pair $M = \langle g, p \rangle$ such that:

- g is any monotone algorithm (for the underlying OP problem)

- $p_i(r) = h_i(r_{-i}) + r_i w_i(r) - \int_0^{r_i} w_i(r_{-i}, z) dz$

where $h_i(r_{-i})$ is an arbitrary function *independent of* r_i.

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To simplify notation we will write $w_i(r)$ in place of $w_i(g(r))$.

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(This will produce negative utilities)

Proof (cont.):

- When $r_i = t_i$:

$$u_i(t_i, g(r_{-i}, t_i)) = p_i(r_{-i}, t_i) - v_i(t_i, g(r_{-i}, t_i))$$

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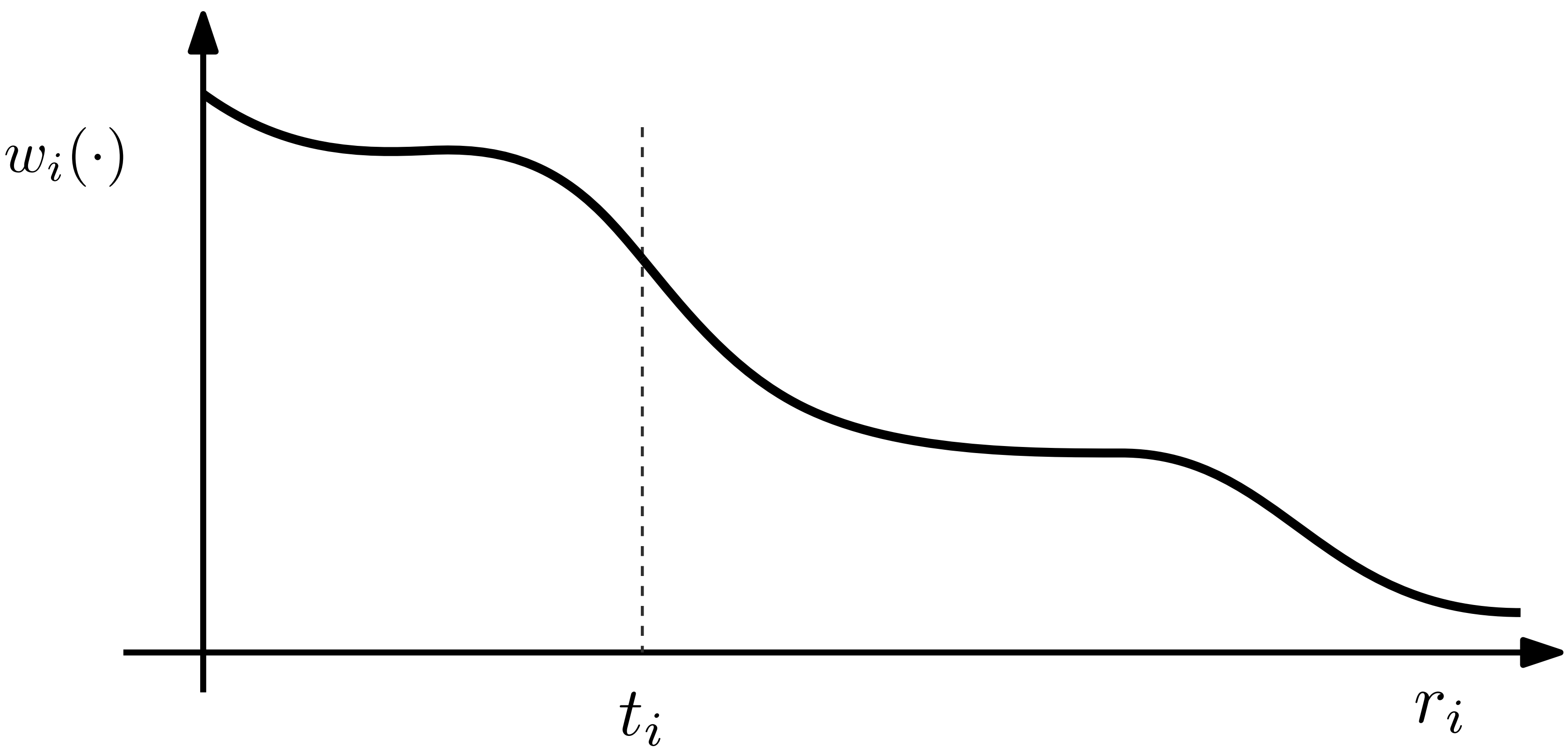

Proof (cont.):

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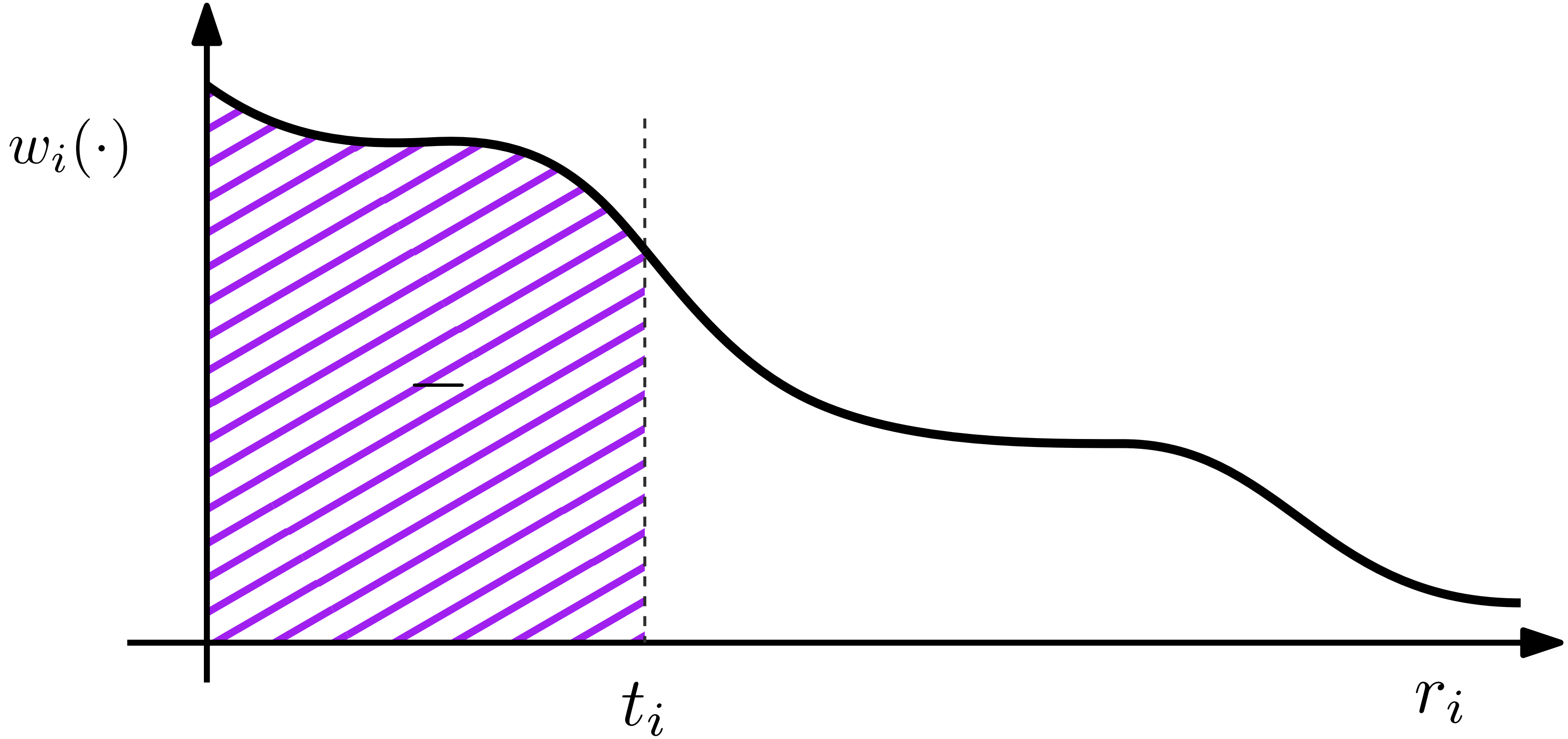

Proof (cont.):

- When $r_i = t_i$: $u_i(t_i, g(r_{-i}, t_i)) = -\int_0^{t_i} w_i(r_{-i}, z)dz$
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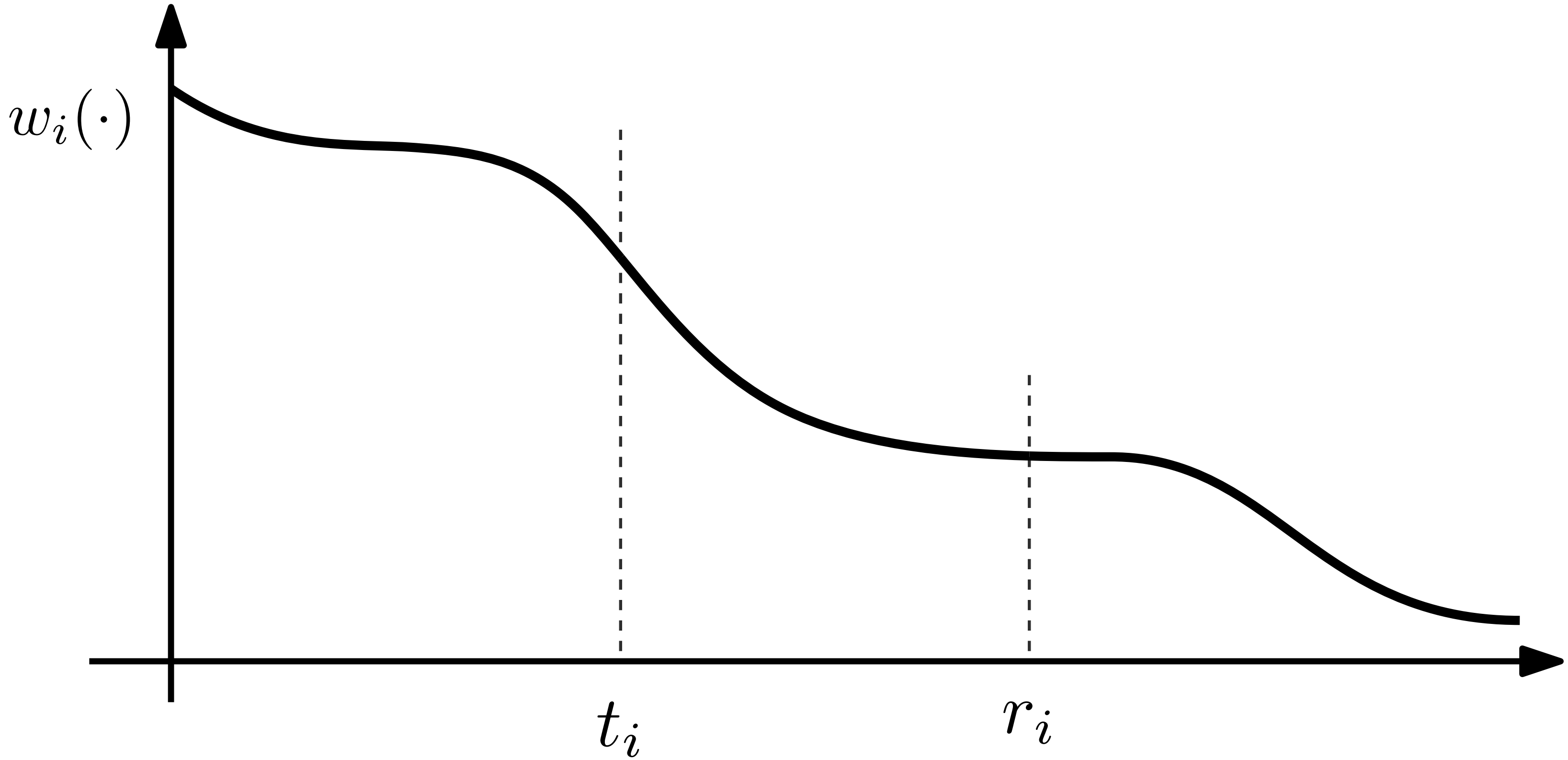

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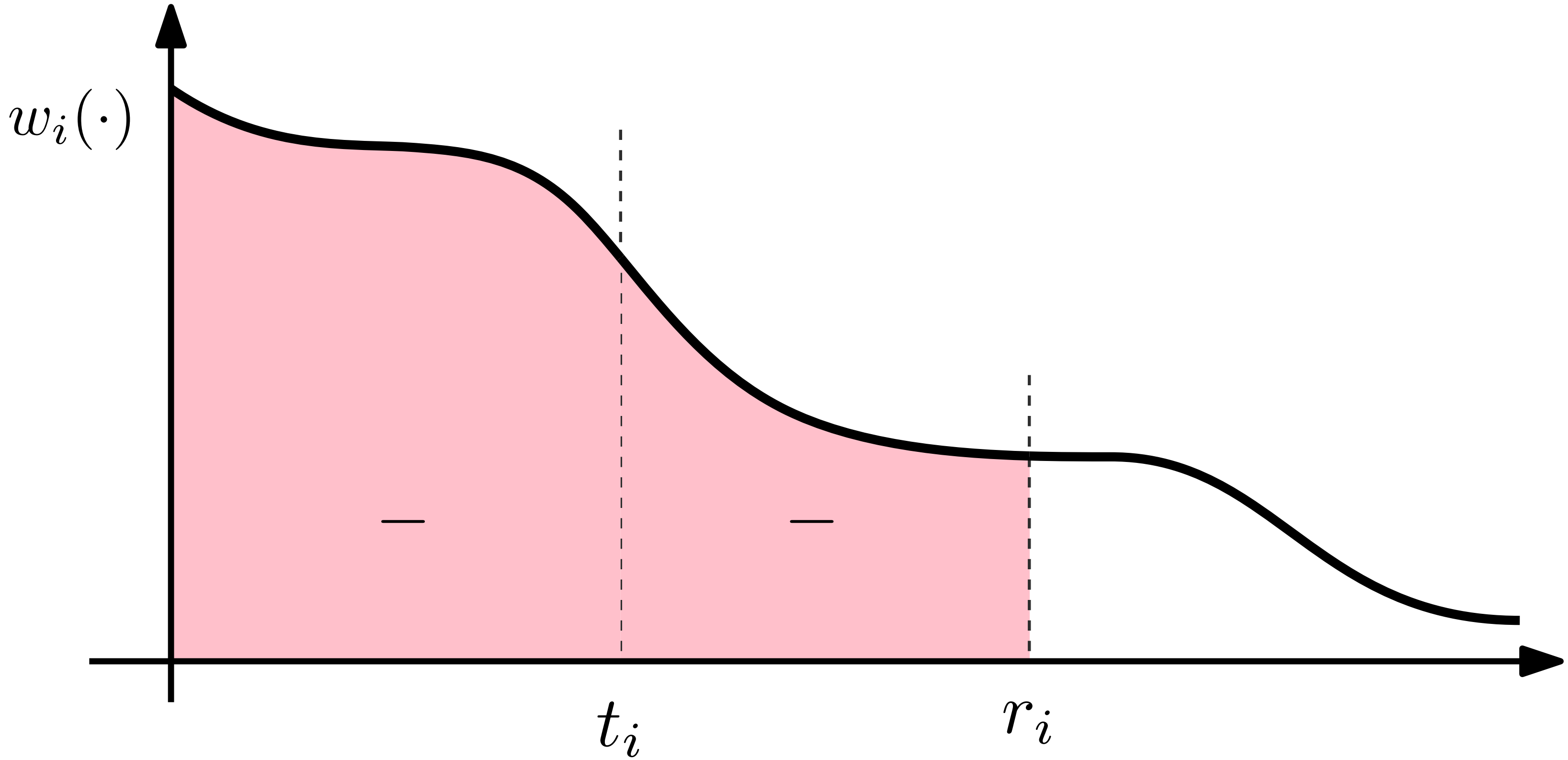

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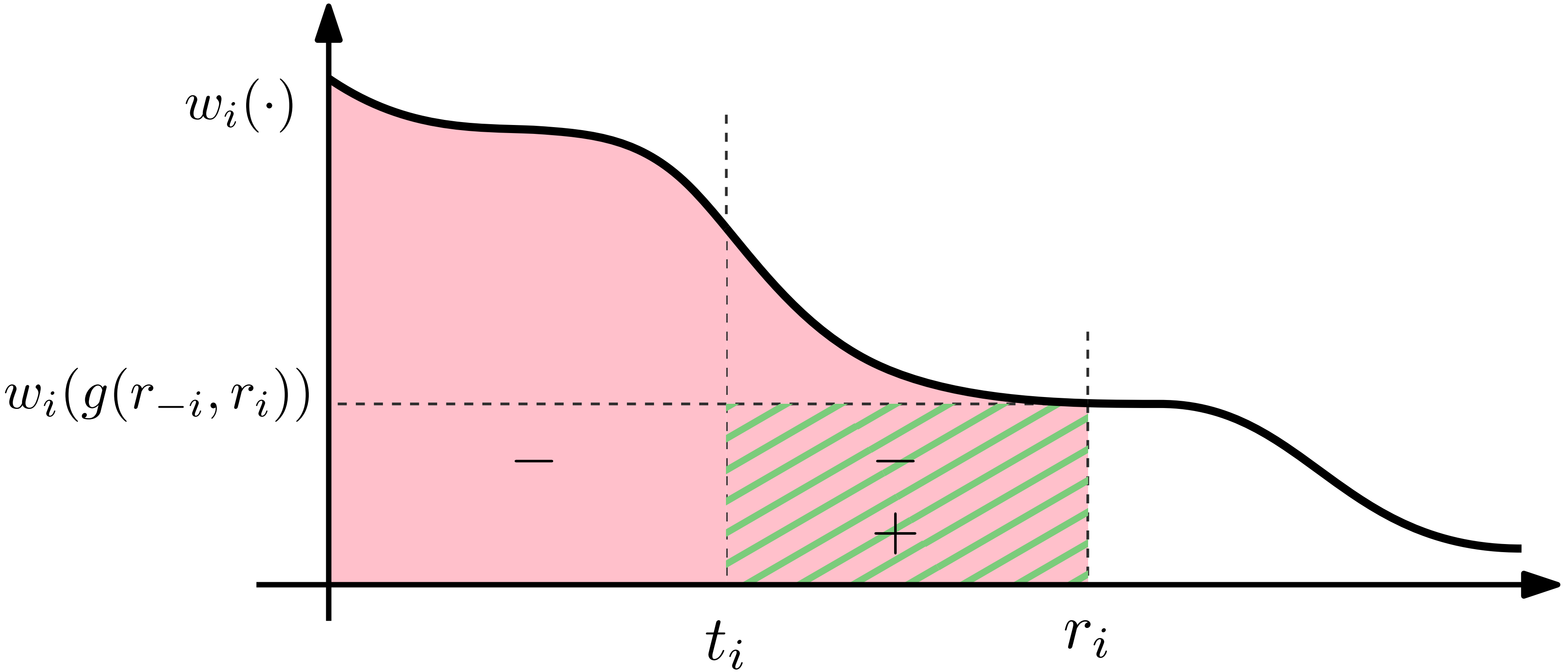

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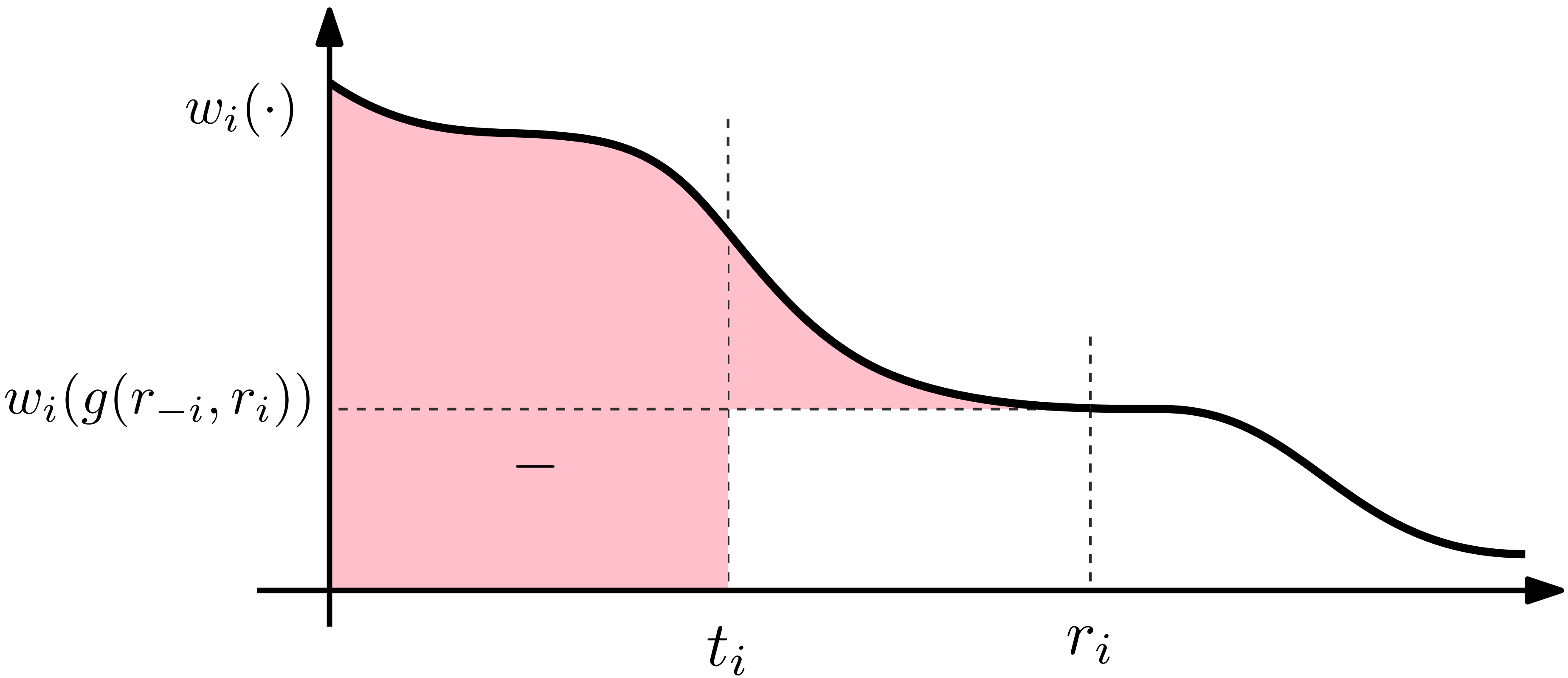

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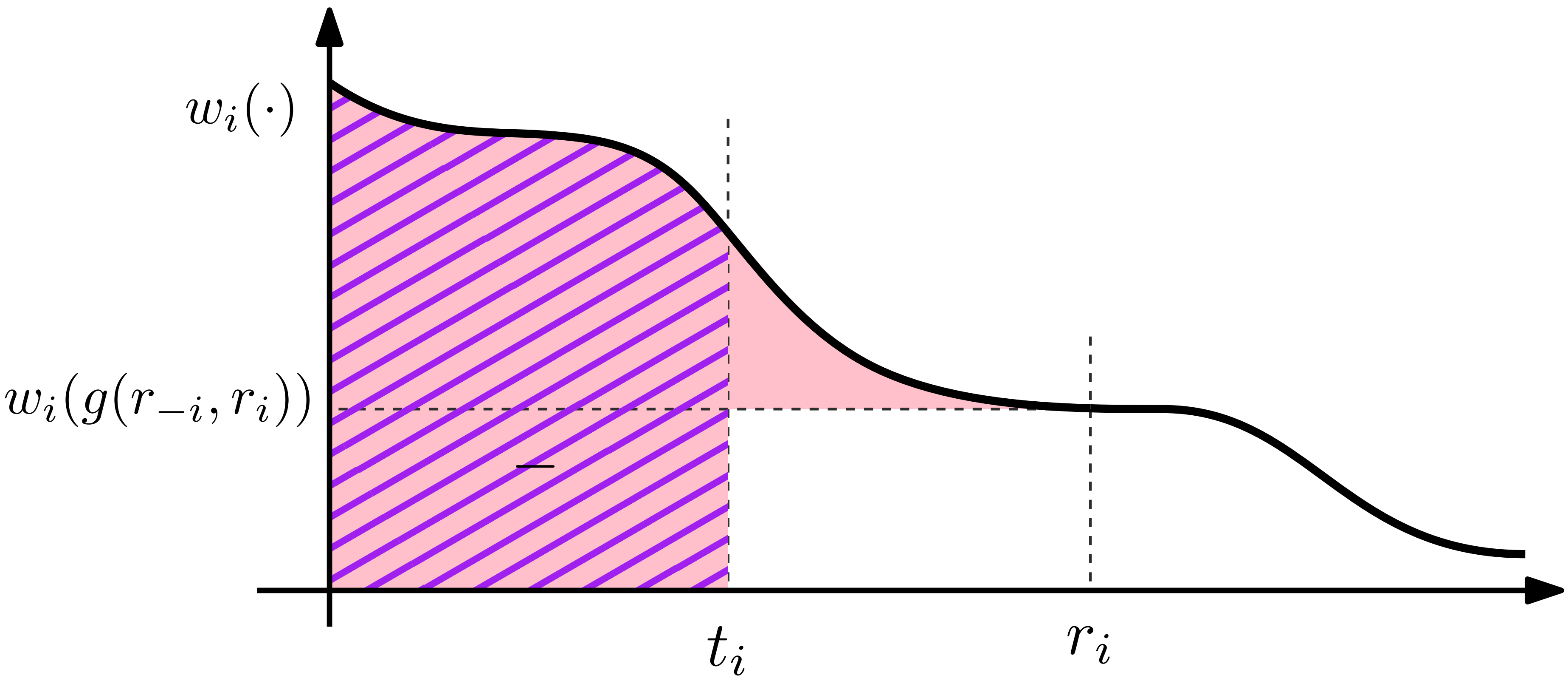

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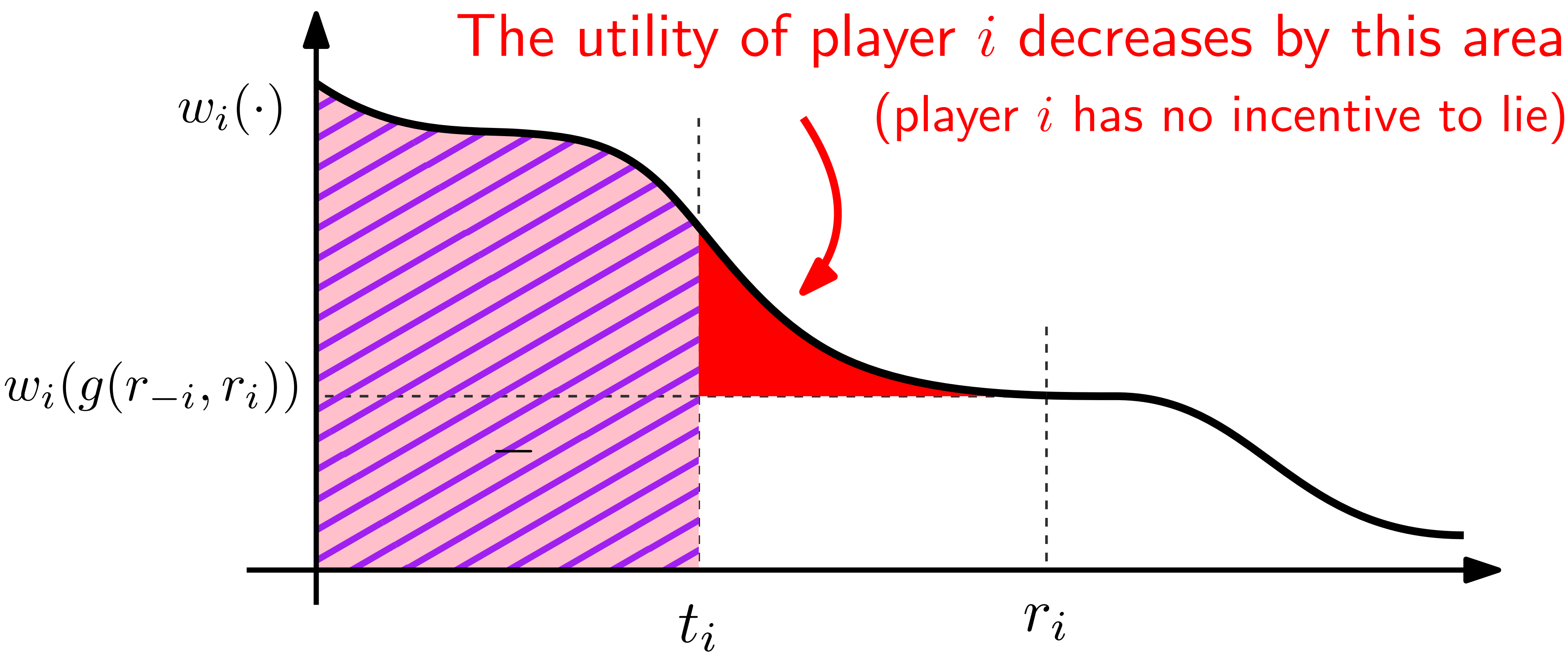

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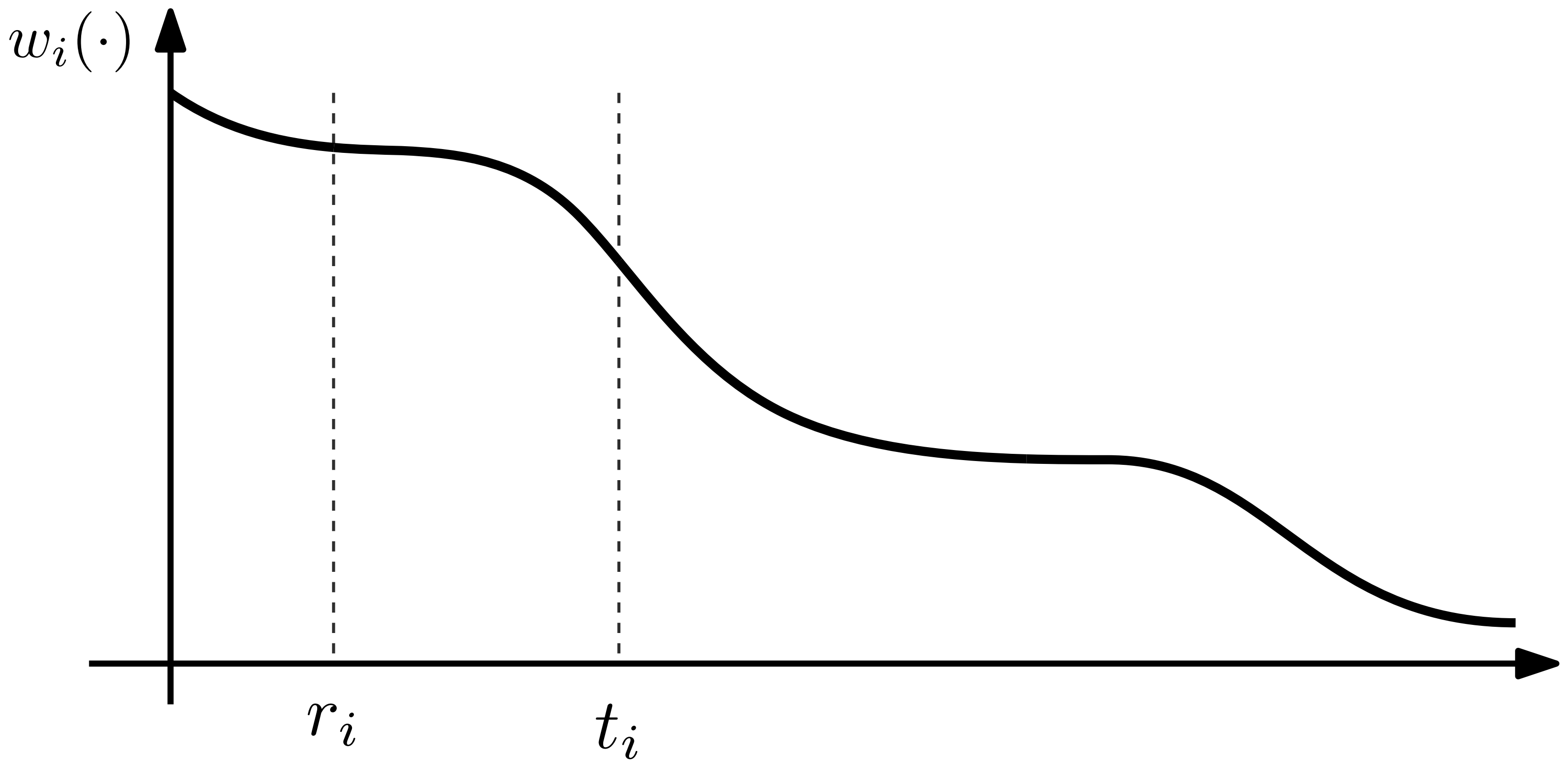

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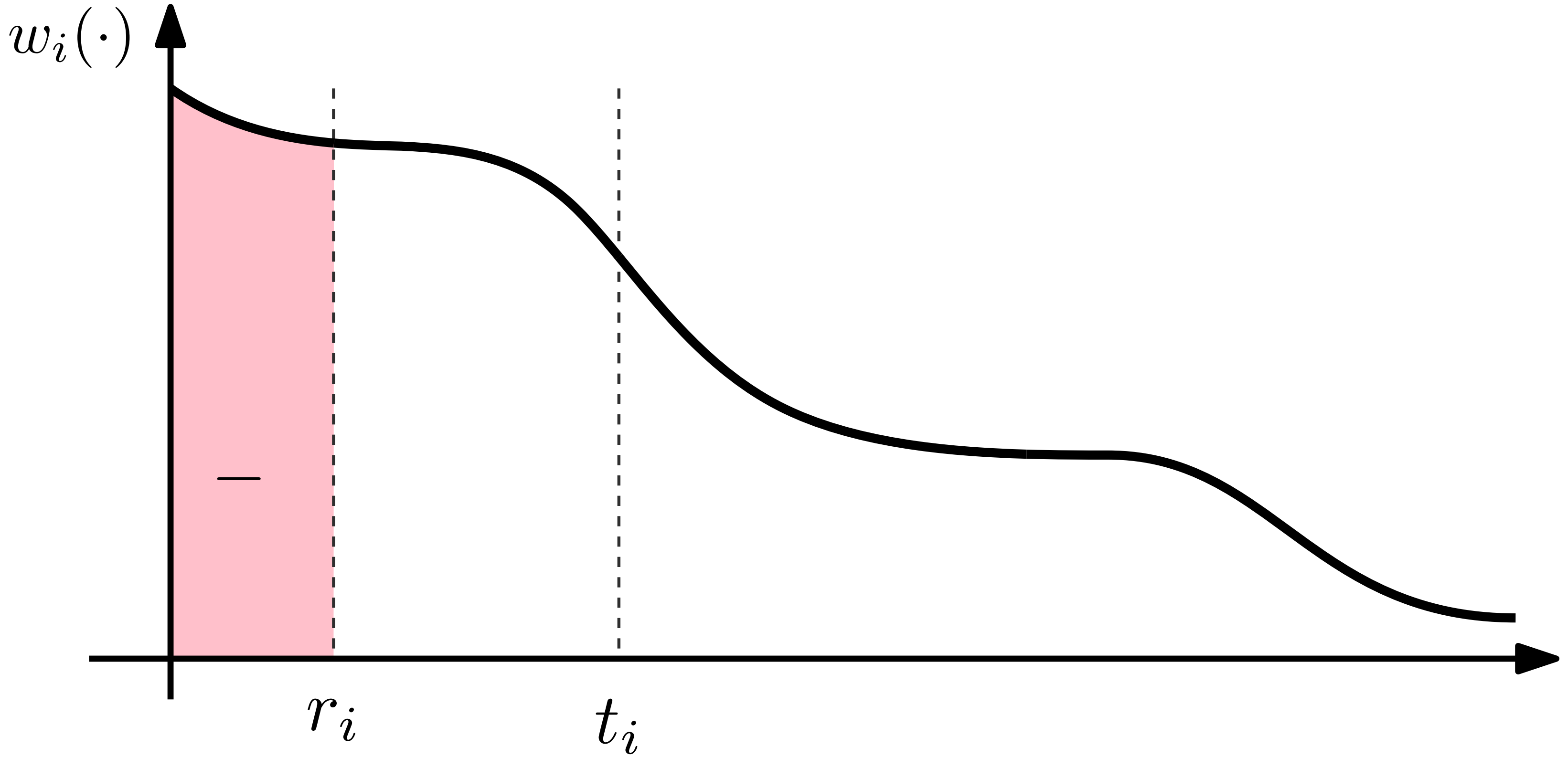

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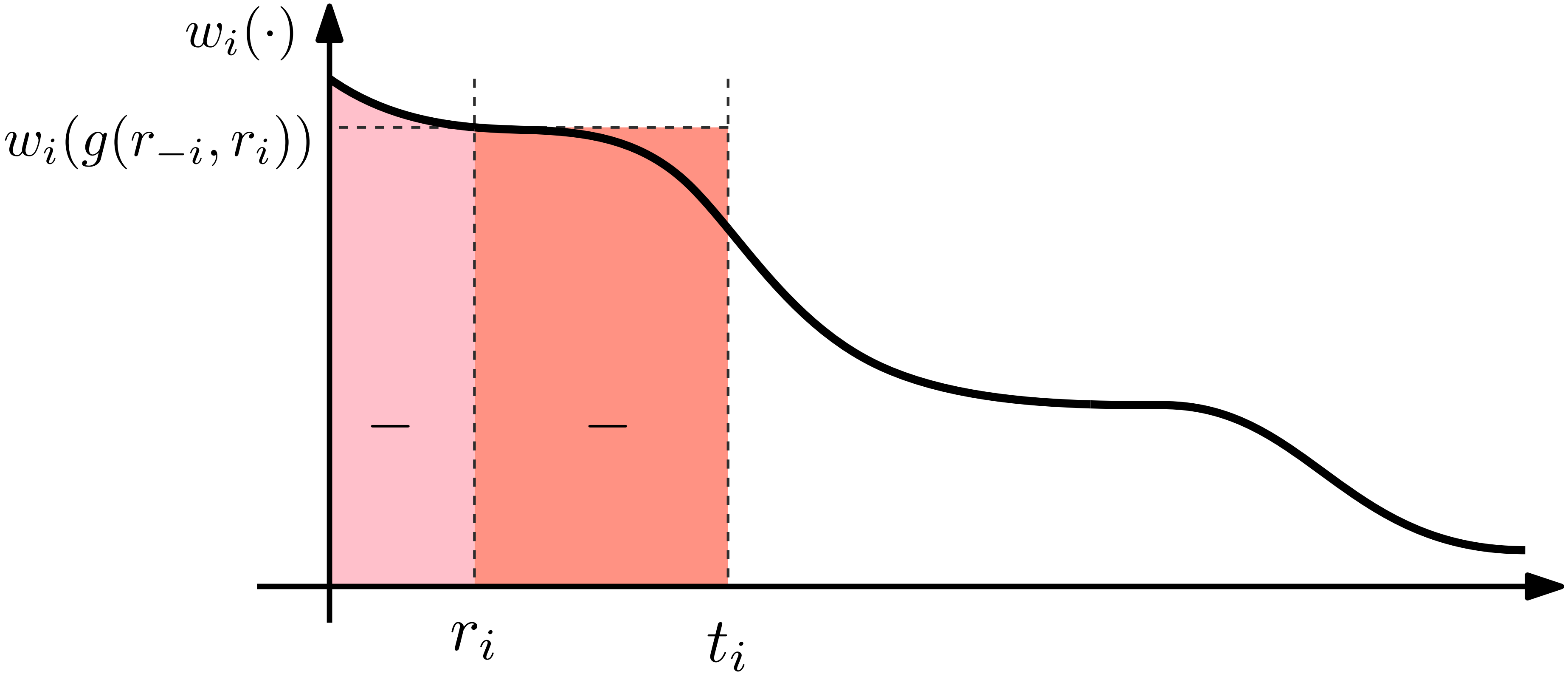

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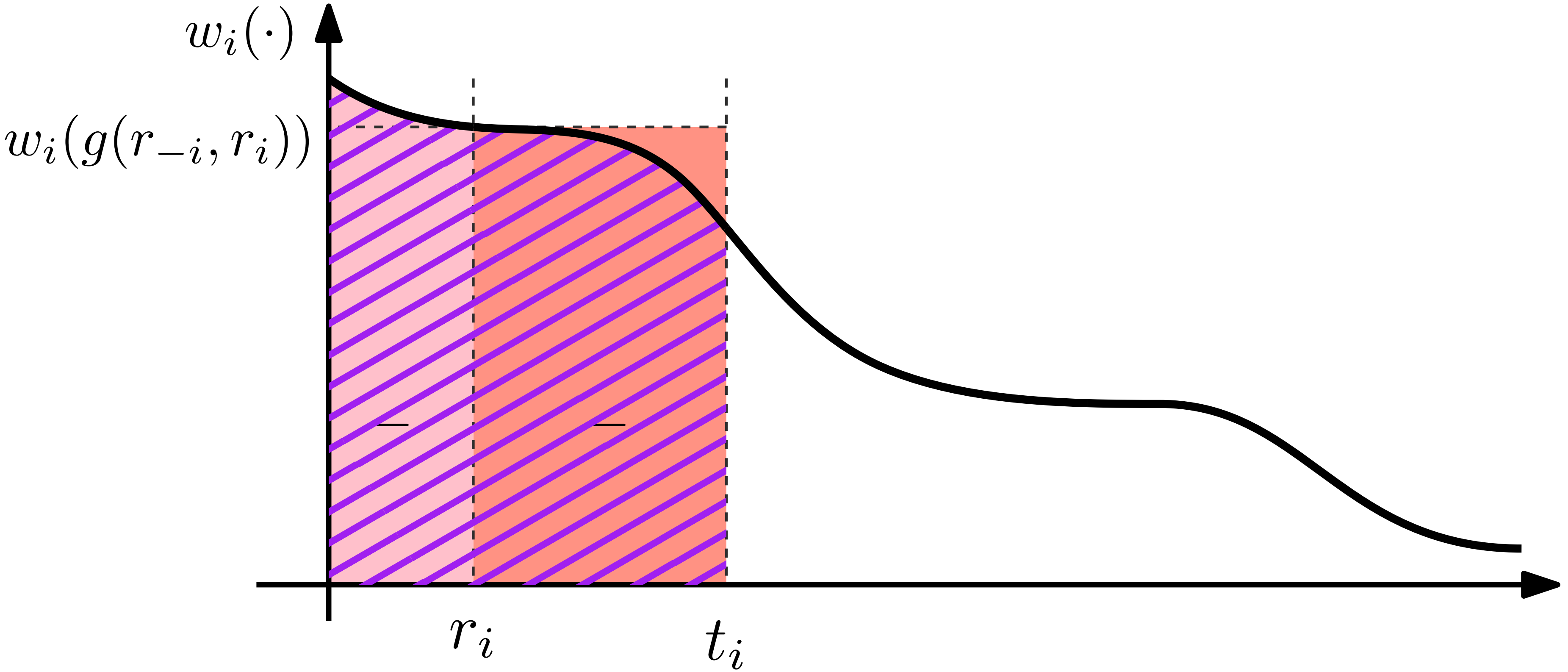

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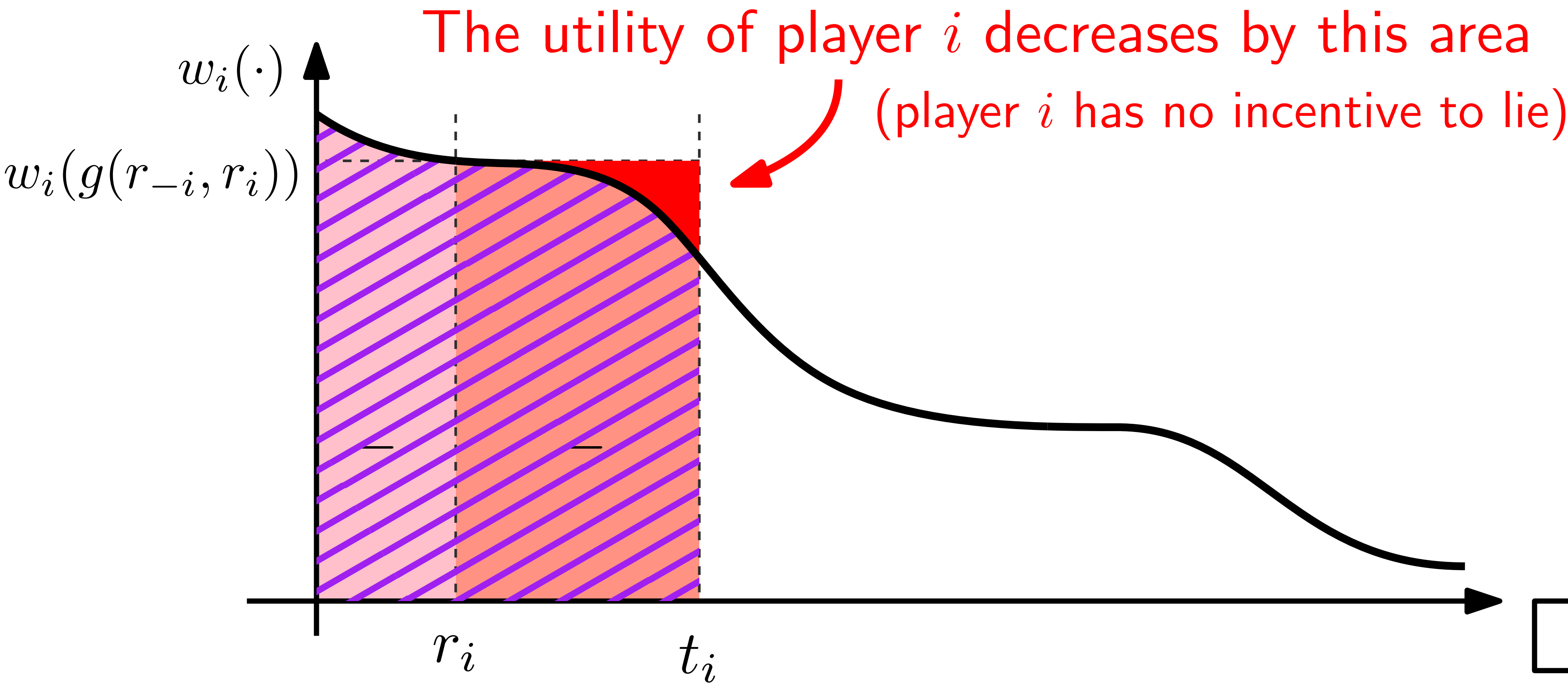
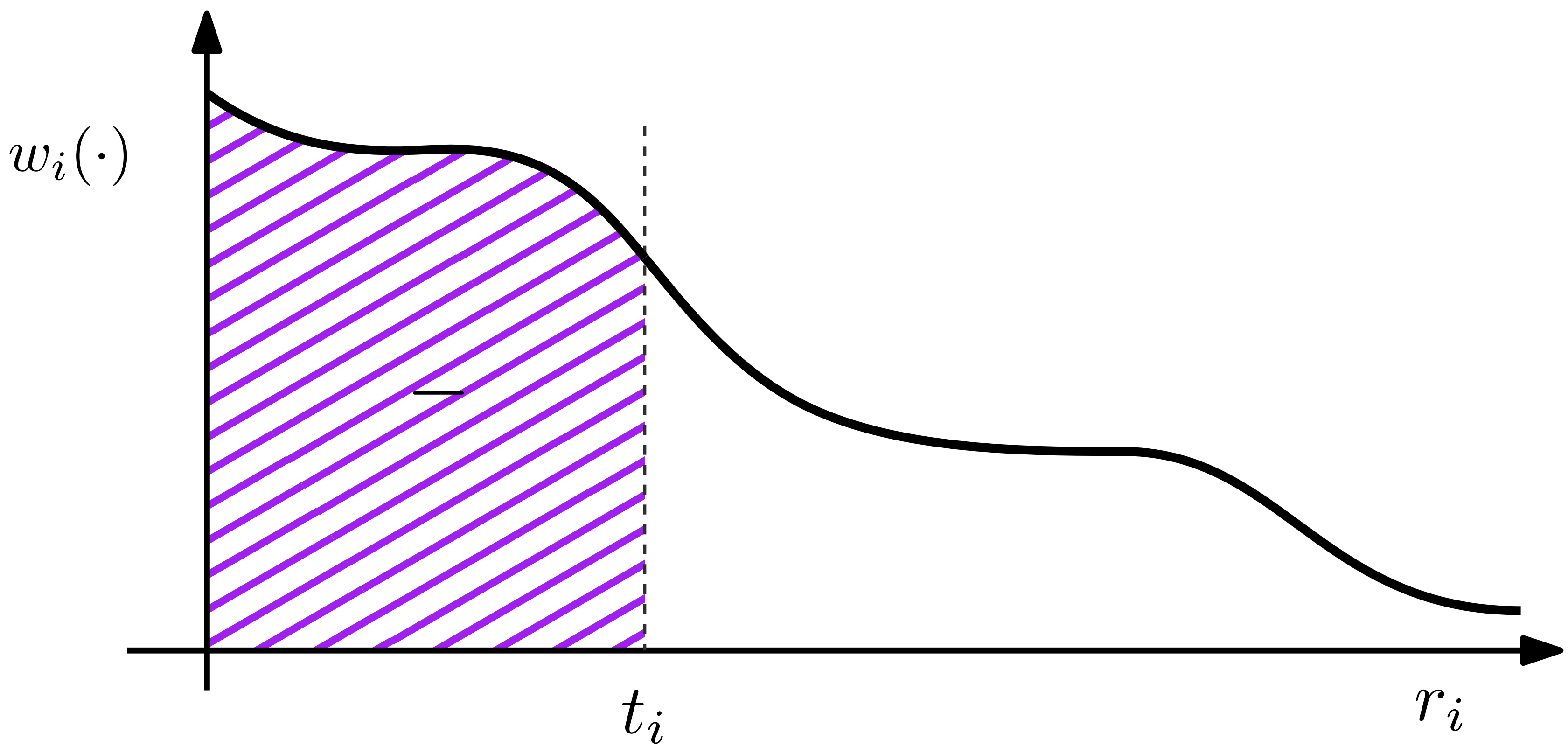


$\square$

Voluntary participation

With $h_i(r_{-i}) = 0$ the mechanism does not guarantee voluntary participation.

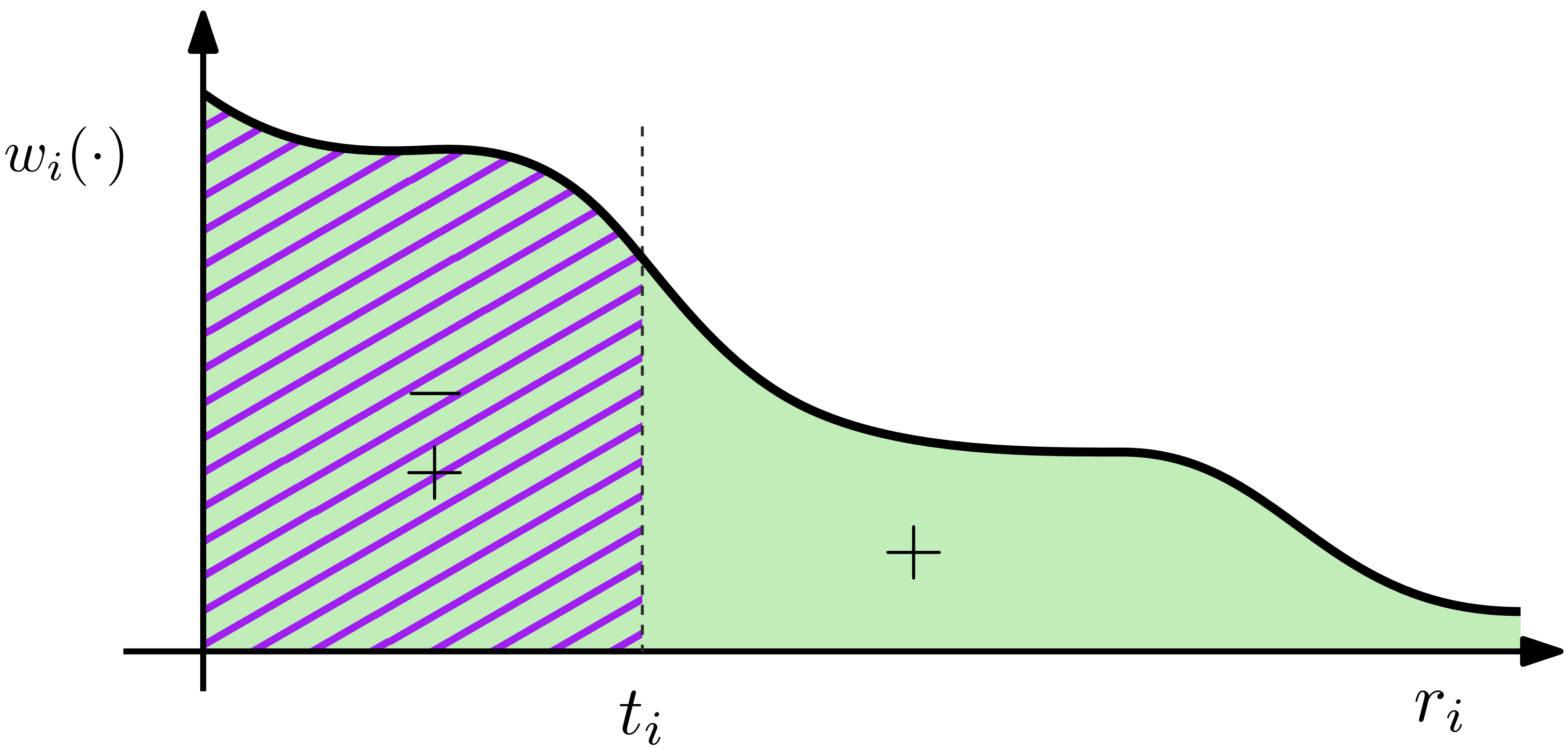
$$u_i(t_i, g(r_{-i}, t_i)) = \underbrace{h_i(r_{-i})}_{0} - \int_0^{t_i} w_i(r_{-i}, z)dz$$



Voluntary participation

Solution: Choose $h_i(r_{-i}) = \int_0^{+\infty} w_i(r_{-i}, z)dz$

$$u_i(t_i, g(r_{-i}, t_i)) = h_i(r_{-i}) - \int_0^{t_i} w_i(r_{-i}, z)dz$$



Voluntary participation

Solution: Choose $h_i(r_{-i}) = \int_0^{+\infty} w_i(r_{-i}, z)dz$

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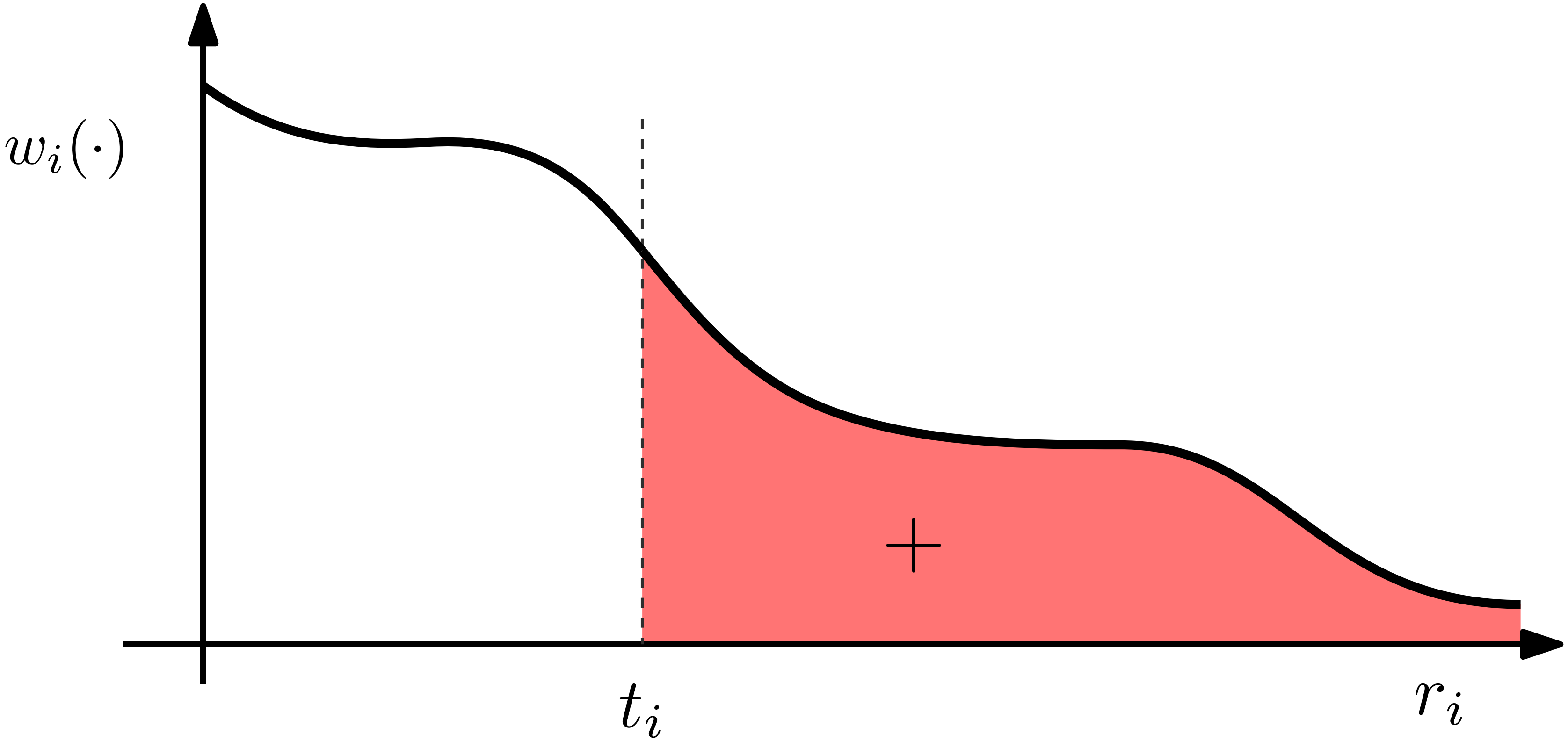

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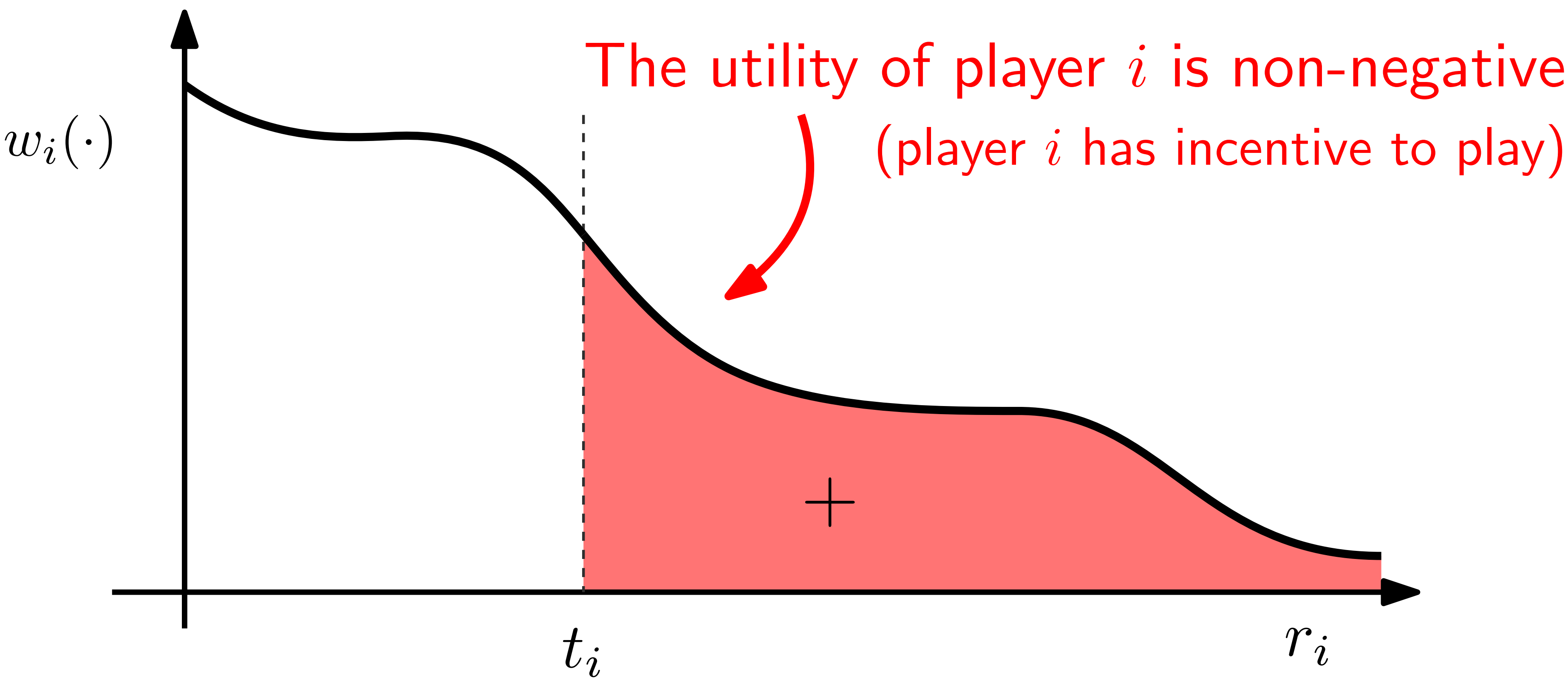

Wrapping up

Truthful One-parameter mechanism that guarantees voluntary participation (for an OP problem):

$$M = \langle g, p \rangle$$

- g is any monotone algorithm (for the underlying OP problem)
- $p_i(r) = r_i w_i(r) + \int_{r_i}^{+\infty} w_i(r_{-i}, z) dz$

VCG vs One-parameter

VCG mechanisms: arbitrary valuation functions and types, but only utilitarian problems

One-parameter mechanisms: arbitrary social-choice function, but only one-parameter types and workloaded valuation functions

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$$\Downarrow$$

The VCG and the OP mechanisms coincide