# Università degli Studi dell'Aquila

**Non-Cooperative Networks: Mid-term Evaluation**

Wednesday, November 7th, 2018 – Prof. Guido Proietti

| Write your data ⟹ | Last name: ................. First name: ........... ID number: ................. | Points |
|---|---|---|
| EXERCISE 1 | | |
| EXERCISE 2 | | |
| TOTAL | | |

**EXERCISE 1: Multiple-choice questions (20 points)**

**Remark:** Only one choice is correct. Use the enclosed grid to select your choice. A correct answer scores 3 points, while a wrong answer receives a $-1$ penalization. You are allowed to omit an answer. If you wrongly select an answer, just make a circle around the wrong $\times$ (i.e., in the following way $\otimes$) and select through a $\times$ the newly selected answer. A question collecting more than one answer will be considered as omitted. The final score will be given by summing up all the obtained points (0 for a missing answer), and then normalizing to 20.

1. A *Dominant Strategy Equilibrium* is a strategy combination $s^* = (s_1^*, \dots, s_N^*)$, such that (assume $p_i$ is a cost):
a) there exists a player $i$ and an alternative strategy profile $s = (s_1, \dots, s_i, \dots, s_N)$, such that $p_i(s_1, \dots, s_i^*, \dots, s_N) \geq p_i(s_1, \dots, s_i, \dots, s_N)$
*b) for each player $i$ and for any possible alternative strategy profile $s = (s_1, \dots, s_i, \dots, s_N)$, $p_i(s_1, \dots, s_i^*, \dots, s_N) \leq p_i(s_1, \dots, s_i, \dots, s_N)$
c) there exist no player $i$ and no alternative strategy profile $s = (s_1, \dots, s_i, \dots, s_N)$, such that $p_i(s_1, \dots, s_i^*, \dots, s_N) \leq p_i(s_1, \dots, s_i, \dots, s_N)$
d) for each player $i$ and for any possible alternative strategy profile $s = (s_1, \dots, s_i, \dots, s_N)$, $p_i(s_1^*, \dots, s_i^*, \dots, s_N^*) \geq p_i(s_1, \dots, s_i, \dots, s_N)$
2. A *Nash Equilibrium* is a strategy combination $s^* = (s_1^*, \dots, s_N^*)$, such that (assume $p_i$ is a utility):
a) there exists a player $i$ and an alternative strategy profile $s = (s_1, \dots, s_i, \dots, s_N)$, such that $p_i(s_1, \dots, s_i^*, \dots, s_N) \leq p_i(s_1, \dots, s_i, \dots, s_N)$
b) for each player $i$ and for any possible alternative strategy profile $s = (s_1, \dots, s_i, \dots, s_N)$, $p_i(s_1, \dots, s_i^*, \dots, s_N) \geq p_i(s_1, \dots, s_i, \dots, s_N)$
c) there exist no player $i$ and no alternative strategy profile $s = (s_1, \dots, s_i, \dots, s_N)$, such that $p_i(s_1, \dots, s_i^*, \dots, s_N) \leq p_i(s_1, \dots, s_i, \dots, s_N)$
*d) for each player $i$ and for any alternative strategy $s_i$ of $i$, $p_i(s_1^*, \dots, s_i^*, \dots, s_N^*) \geq p_i(s_1^*, \dots, s_i, \dots, s_N^*)$
3. How the Price of Anarchy is defined for a game in which the social choice function $C$ has to be minimized ($S$ is the set of Nash equilibria)?
*a) $\text{PoA} = \sup_{s \in S} \frac{C(s)}{C(\text{OPT})}$ b) $\text{PoA} = \inf_{s \in S} \frac{C(s)}{C(\text{OPT})}$ c) $\text{PoA} = \sup_{s \in S} \frac{C(\text{OPT})}{C(s)}$ d) $\text{PoA} = \inf_{s \in S} \frac{C(\text{OPT})}{C(s)}$
4. How the Price of Stability is defined for a game in which the social-choice function $C$ has to be maximized ($S$ is the set of Nash equilibria)?
*a) $\text{PoS} = \sup_{s \in S} \frac{C(s)}{C(\text{OPT})}$ b) $\text{PoS} = \inf_{s \in S} \frac{C(s)}{C(\text{OPT})}$ c) $\text{PoS} = \sup_{s \in S} \frac{C(\text{OPT})}{C(s)}$ d) $\text{PoS} = \inf_{s \in S} \frac{C(\text{OPT})}{C(s)}$
5. In a network with $k$ players and degree-$p$ polynomial latency functions, which of the following claim on the selfish routing game is true?
a) The PoA is at most $4/3$ b) The PoA is at most $p$ *c) The PoA is $O(p/\log p)$ d) The PoA is at most $k$, and this is tight
6. In the global connection game with $k$ players on a graph $G = (V, E, c)$, if we denote by $c_e$ (resp., $k_e$) the cost (resp., the load) of an edge $e \in E$, and by $N(S)$ the network induced by a given strategy profile $S$, which of the following claim is false?
a) $\Psi(S) = \sum_{e \in N(S)} c_e \cdot (1 + 1/2 + \dots 1/k_e)$ is a potential function b) Finding a best response for a player is polynomial
c) The PoA is at most $k$, and this is tight * d) The PoS is at most $H_k$, the $k$-th harmonic number, but this is not tight
7. In a local connection game with $k$ players and building cost $\alpha \geq 0$, which of the following claim is false?
* a) for $\alpha \geq 1$, the star is an optimal solution b) for $\alpha = 1$, the clique and the star are stable graphs c) $\text{PoA} \leq 6\sqrt{\alpha} + 3$ d) $\text{PoS} \leq 4/3$
8. In the Malik, Mittal and Gupta algorithm on a graph with $n$ nodes and $m$ edges, which of the following set of operations are performed on the Fibonacci heap?
a) A single make-heap, $O(n)$ insert, $n$ find-min, $O(n)$ delete and $O(m)$ decrease-key
b) A single make-heap, $n$ insert, $O(n)$ find-min, $n$ delete and $O(m)$ decrease-key
*c) A single make-heap, $n$ insert, $O(n)$ find-min, $O(n)$ delete and $O(m)$ decrease-key
d) A single make-heap, $n$ insert, $O(n)$ find-min, $O(n)$ delete and $m$ decrease-key
9. Which of the following corresponds to the definition on the inverse of the Ackermann function?
a) $\alpha(m, n) = \min\{i \geq 1 | A(i, \lfloor m/n \rfloor) \geq n\}$ b) $\alpha(m, n) = \min\{i \geq 1 | A(\lceil m/n \rceil, i) \geq \log n\}$
*c) $\alpha(m, n) = \min\{i \geq 1 | A(i, \lfloor m/n \rfloor) \geq \log n\}$ d) $\alpha(m, n) = \min\{i \geq 1 | A(i, \lceil m/n \rceil) \geq \log n\}$
10. In the selfish-edge single-source shortest-path tree problem, which of the following corresponds to the threshold value for an edge $e = (u, v)$ belonging to the solution?
*a) $\Theta_e = \min_{f=(x,y) \in C(e)} \{d_G(s, x) + r(e) + d_G(y, v)\} - d_G(s, u)$ b) $\Theta_e = \min_{f=(x,y) \in C(e)} \{d_G(s, x) + r(e) + d_G(y, v)\} - d_G(s, v)$
c) $\Theta_e = \min_{f=(x,y) \in C(e)} \{d_{G-e}(s, x) + r(e) + d_{G-e}(y, v)\} - d_G(s, u)$ d) $\Theta_e = \min_{f=(x,y) \in C(e)} \{d_{G-e}(s, x) + r(e) + d_{G-e}(y, v)\} - d_G(s, v)$

**Answer Grid**

| | Question | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Choice | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| a | | | | | | | | | | |
| b | | | | | | | | | | |
| c | | | | | | | | | | |
| d | | | | | | | | | | |

**EXERCISE 2: Open question (10 points)**

**Remark:** Select at your choice one out of the following two questions, and address it exhaustively.

1. Describe and analyze the local connection game.
2. Describe and analyze the VCG-mechanism for the single-edge MST problem.