



# UNIVERSITÀ DEGLI STUDI DELL'AQUILA

## Non-Cooperative Networks: Mid-term Evaluation

Tuesday, November 7th, 2017 – Prof. Guido Proietti

|                    |                  |                   |                  |        |
|--------------------|------------------|-------------------|------------------|--------|
| Write your data => | Last name: ..... | First name: ..... | ID number: ..... | Points |
| EXERCISE 1         |                  |                   |                  |        |
| EXERCISE 2         |                  |                   |                  |        |
| TOTAL              |                  |                   |                  |        |

### EXERCISE 1: Multiple-choice questions (20 points)

**Remark:** Only one choice is correct. Use the enclosed grid to select your choice. A correct answer scores 3 points, while a wrong answer receives a  $-1$  penalization. You are allowed to omit an answer. If you wrongly select an answer, just make a circle around the wrong  $\times$  (i.e., in the following way  $\otimes$ ) and select through a  $\times$  the newly selected answer. A question collecting more than one answer will be considered as omitted. The final score will be given by summing up all the obtained points (0 for a missing answer), and then normalizing to 20.

- A *Dominant Strategy Equilibrium* is a strategy combination  $s^* = (s_1^*, \dots, s_N^*)$ , such that (assume  $p_i$  is a utility):
  - there exists a player  $i$  and an alternative strategy profile  $s = (s_1, \dots, s_i, \dots, s_N)$ , such that  $p_i(s_1, \dots, s_i^*, \dots, s_N) \leq p_i(s_1, \dots, s_i, \dots, s_N)$
  - for each player  $i$  and for any possible alternative strategy profile  $s = (s_1, \dots, s_i, \dots, s_N)$ ,  $p_i(s_1, \dots, s_i^*, \dots, s_N) \geq p_i(s_1, \dots, s_i, \dots, s_N)$
  - there exist no player  $i$  and no alternative strategy profile  $s = (s_1, \dots, s_i, \dots, s_N)$ , such that  $p_i(s_1, \dots, s_i^*, \dots, s_N) \geq p_i(s_1, \dots, s_i, \dots, s_N)$
  - for each player  $i$  and for any possible alternative strategy profile  $s = (s_1, \dots, s_i, \dots, s_N)$ ,  $p_i(s_1^*, \dots, s_i^*, \dots, s_N^*) \leq p_i(s_1, \dots, s_i, \dots, s_N)$
- A *Nash Equilibrium* is a strategy combination  $s^* = (s_1^*, \dots, s_N^*)$ , such that (assume  $p_i$  is a cost):
  - there exists a player  $i$  and an alternative strategy profile  $s = (s_1, \dots, s_i, \dots, s_N)$ , such that  $p_i(s_1, \dots, s_i^*, \dots, s_N) \geq p_i(s_1, \dots, s_i, \dots, s_N)$
  - for each player  $i$  and for any possible alternative strategy profile  $s = (s_1, \dots, s_i, \dots, s_N)$ ,  $p_i(s_1, \dots, s_i^*, \dots, s_N) \leq p_i(s_1, \dots, s_i, \dots, s_N)$
  - there exist no player  $i$  and no alternative strategy profile  $s = (s_1, \dots, s_i, \dots, s_N)$ , such that  $p_i(s_1, \dots, s_i^*, \dots, s_N) \geq p_i(s_1, \dots, s_i, \dots, s_N)$
  - for each player  $i$  and for any alternative strategy  $s_i$  of  $i$ ,  $p_i(s_1^*, \dots, s_i^*, \dots, s_N^*) \leq p_i(s_1^*, \dots, s_i, \dots, s_N^*)$
- How the Price of Anarchy is defined for a game in which the social choice function  $C$  has to be maximized ( $S$  is the set of Nash equilibria)?
  - $\text{PoA} = \sup_{s \in S} \frac{C(s)}{C(\text{OPT})}$
  - $\text{PoA} = \inf_{s \in S} \frac{C(s)}{C(\text{OPT})}$
  - $\text{PoA} = \sup_{s \in S} \frac{C(\text{OPT})}{C(s)}$
  - $\text{PoA} = \inf_{s \in S} \frac{C(\text{OPT})}{C(s)}$
- How the Price of Stability is defined for a game in which the social-choice function  $C$  has to be minimized ( $S$  is the set of Nash equilibria)?
  - $\text{PoS} = \sup_{s \in S} \frac{C(s)}{C(\text{OPT})}$
  - $\text{PoS} = \inf_{s \in S} \frac{C(s)}{C(\text{OPT})}$
  - $\text{PoS} = \sup_{s \in S} \frac{C(\text{OPT})}{C(s)}$
  - $\text{PoS} = \inf_{s \in S} \frac{C(\text{OPT})}{C(s)}$
- In a network with  $k$  players and linear latency functions, which of the following claim on the selfish routing game is true?
  - The PoS is at least  $4/3$
  - The PoS is at most 1
  - The PoA is at least  $4/3$
  - The PoA is at most  $k$ , and this is tight
- In the global connection game on a graph  $G = (V, E, c)$ , if we denote by  $c_e$  (resp.,  $k_e$ ) the cost (resp., the load) of an edge  $e \in E$ , and by  $N(S)$  the network induced by a given strategy profile  $S$ , which of the following is a potential function?
  - $\Psi(S) = \sum_{e \in N(S)} c_e \cdot (1 + 1/2 + \dots + 1/k_e)$
  - $\Psi(S) = \sum_{e \in N(S)} c_e/k_e$
  - $\Psi(S) = \sum_{e \in N(S)} c_e$
  - $\Psi(S) = \sum_{e \in N(S)} c_e \cdot (1 + 2 + \dots + k_e)$
- In a local connection game with  $k$  players and building cost  $\alpha \geq 0$ , which of the following claim is true?
  - for  $\alpha = 3/2$ ,  $\text{PoS} = 1$
  - for  $\alpha = 1$ , the clique is the only stable graph
  - $\text{PoA} = O(1)$
  - $\text{PoS} \leq 4/3$
- Which of the following is a Clarke payment scheme?
  - $p_i(g(r)) = \sum_{j \neq i} v_j(r_j, g(r-i)) - \sum_{j \neq i} v_j(r_j, g(r))$
  - $p_i(g(r)) = - \sum_{j \neq i} v_j(r_j, g(r)) + \sum_j v_j(r_j, g(r))$
  - $p_i(g(r)) = - \sum_{j \neq i} v_j(r_j, g(r-i)) + \sum_{j \neq i} v_j(r_j, g(r))$
  - $p_i(g(r)) = - \sum_j v_j(r_j, g(r-i)) + \sum_{j \neq i} v_j(r_j, g(r))$
- In the Malik, Mittal and Gupta algorithm for the selfish-edge shortest path problem, which of the following keys is associated with a node  $y$  in the priority queue when an edge  $e$  of a graph  $G = (V, E)$  is considered?
  - $k(y) = \min_{(x,y) \in E, x \in N_s(e)} \{d_G(s, x) + r(x, y) + d_G(y, z)\}$
  - $k(y) = \max_{(x,y) \in E, x \in M_s(e)} \{d_G(s, x) + r(x, y) + d_G(y, z)\}$
  - $k(y) = \min_{(x,y) \in E, x \in M_s(e)} \{d_G(s, x) + r(x, y) + d_G(y, z)\}$
  - $k(y) = \min_{(x,y) \in E, x \in M_s(e)} \{d_G(s, x) + 1 + d_G(y, z)\}$
- In the one-parameter mechanism for the single-source shortest path tree problem, which payment will receive an edge  $e$  belonging to the solution?
  - $p_e = r_e w_e(r) + \int_{r_e}^{\infty} w_e(r-e, z) dz$
  - $p_e = r_e w_e(r) + \int_0^{\infty} w_e(r-e, z) dz$
  - $p_e = -r_e w_e(r) + \int_{r_e}^{\infty} w_e(r-e, z) dz$
  - $p_e = r_e w_e(r) + \int_0^{r_e} w_e(r-e, z) dz$

### Answer Grid

|        | Question |   |   |   |   |   |   |   |   |    |
|--------|----------|---|---|---|---|---|---|---|---|----|
| Choice | 1        | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| a      |          |   |   |   |   |   |   |   |   |    |
| b      |          |   |   |   |   |   |   |   |   |    |
| c      |          |   |   |   |   |   |   |   |   |    |
| d      |          |   |   |   |   |   |   |   |   |    |

### EXERCISE 2: Open question (10 points)

**Remark:** Select at your choice one out of the following three questions, and address it exhaustively.

- Describe and analyze the selfish routing game.
- Describe and analyze the global connection game.
- Describe and analyze the VCG-mechanism for the single-edge single-pair shortest path problem.